DISTRIBUTION OF HYDRODYNAMIC PRESSURE IN THE INTERSPACE OF LATERAL SLIDING BEARINGS LUBRICATED BY FERROFLUID WITH DIFFERENT CONCENTRATIONS OF MAGNETIC PARTICLES

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Abstract

This paper presents the numerical results of hydrodynamic pressure distributions and capacity in the gap of ferrofluid-lubricated slide bearing with different concentrations of magnetic particles.

Ferrofluid is a colloidal mixture of dispersing agent (base) and diffused (magnetic particles).

The most common distracted factors are Fe_3O_4, Ge_2O_2 or NiO. Scattering factor is mostly water, mineral oil, synthetic oil. In addition, the magnetic particles are coated with a surfactant in the form of long chains of molecules or electrostatic coating.

Reynolds-type equation has been derived from the equations of momentum and continuity equation for laminar, steady and isothermal flow. Viscoelastic model Rivlin-Ericksen type of lubricant has been adopted. It has been adopted also that the dynamic viscosity depends generally on the magnetic field. The equations of momentum, continuity and Maxwell's equation with a constitutive equation were made dimensionless, and then them small units of higher order have been omitted (units of the order of the relative radial clearance: ψ=0.001). Such estimated system of equations of motion was divided into two subsystems by the small parameter method. The basic system of differential equations which consider the effect of the magnetic field has been received and the additional system which takes into account influences of non-Newtonian properties as well.

Integrating the appropriate momentum equations and imposing boundary conditions the velocity vector components and the Reynolds-type equation have been obtained.

While computer simulations of the magnetic field were assumed that the longitudinal component of magnetic induction vector does not change with the length of the bearing.

Reynolds-type equation by which the hydrodynamic pressure distributions can be determined has been solved numerically using Mathcad and ourselves calculation procedures.

The paper presents results of numerical calculations of hydrodynamic pressure distribution and lift force for the simulated values of the magnetic field and ferrofluid's dynamic viscosity and for different relative eccentricities and dimensionless length of the bearing.

The results presented in the paper are the solutions of a basic system.

Keywords: *hydrodynamic pressure distributions, capacity, ferrofluid, magnetic field, numeric calculation*

1. Introduction

The operation of sliding bearings in the absence of gravity, vacuums, or in case of strong magnetic or radioactive fields faces serious problems when using conventional lubricating agents. Some problems with maintaining fluid lubrication also appear in the case of sliding friction loaded in wide-ranged forces. In these cases a ferrofluid can be used as lubricant. Maintenance of lubricant in the oil-gap as well as the viscosity change occurs through controlling of the external magnetic field. The intensity of the lubricant's impact in the bearing gap can be controlled by changing the concentration of magnetic particles or by the changing the intensity of external magnetic field.

There is the task of this paper to present the effect of concentration of magnetic particles on the hydrodynamic pressure distribution and capacity.

In order to make numerical calculations of distribution of hydrodynamic pressure and hydrodynamic lift, the Reynolds-type equation has been derived from the ground up with the fundamental equations namely equations of momentum and equations of stream's continuity. There have been also used Maxwell's equations for the ferrofluid in the case of stationary magnetic field's existence. It has been assumed stationary and laminar flow of lubricant liquid and the isothermal model for lubrication of sliding bearings. The constitutive equation has been used Rivlin-Ericksen one. The cylindrical journal bearing of finite length with the smooth sleeve of whole angle of a belt has been taken into consideration.

In a thin layer of oil film has been assumed constancy of the oil density with temperature changes and the independence of the oil's thermal conductivity coefficient from thermal changes. The viscosity of the oil depends mainly on the magnetic field.

2. Basic equations

Analysis of magnetohydrodynamic lubrication of the cross sliding bearings for a stationary, laminar, isothermal flow involves the solution of fundamental equations, namely equations of conservation of momentum and continuity of the stream in the following forms [1-3, 6, 7]:

$$0 = Div\ \boldsymbol{S} + \mu_o(\boldsymbol{N} \cdot \nabla)\boldsymbol{H} + \tfrac{1}{2}\mu_o rot(\boldsymbol{N} \times \boldsymbol{H}), \tag{1}$$

$$div(\rho \boldsymbol{v}) = 0, \tag{2}$$

$$\text{rot H} = 0, \tag{3}$$

$$\text{div B} = 0, \tag{4}$$

$$B = \mu o(H+N),\ N = H \cdot \chi, \tag{5}$$

where:

B - vector of magnetic induction in ferrofluid [T],
H - vector of magnetic field strength in ferrofluid [$A \cdot m^{-1}$],
N - vector of ferrofluid magnetization [$A \cdot m^{-1}$],
S - ferrofluid stress tensor with coordinates τ_{ij} for i,j=ϕ,r,z [Pa],
v - ferrofluid velocity vector [$m \cdot s^{-1}$],
∇ - Nabila's operator,
μ_o - vacuum magnetic permeability [$H \cdot m^{-1}$],
ρ - density of ferrofluid [$kg \cdot m^{-3}$],
χ - magnetic susceptibility factor of ferrofluid.

Rivlin-Ericksen formula describing the relationship between the coordinates of the stress tensor $\mathbf{S} \equiv \|\tau_{ij}\|$ and the strain rate tensor coordinates of ferrofluid can be presented in the following form [7, 5, 8]:

$$S = {}^{-}p\ I + {}^{\eta}A1 + {}^{\alpha}A1A1 + {}^{\beta}A2. \tag{6}$$

Strain rate tensors can be defined by the following relation [7]:

$$A1 \equiv L + LT, \tag{7}$$

$$A2 \equiv \text{grad } a + (\text{grad } a)T + 2LT \cdot L, \tag{8}$$

where the acceleration vector is given by:

$$a \equiv L \cdot v, \tag{9}$$

$$L \equiv \text{grad } v, \tag{10}$$

where:

$\mathbf{A}_1$ - the first one strain rate tensor [s^{-1}],

A$_2$ - the second one strain rate tensor [s^{-2}],
I - elementary tensor,
L - tensor of gradient taken from the velocity vector [s^{-1}],
a - accelerating vector [$m \cdot s^{-2}$],
p - hydrodynamic pressure [Pa],
α, β - the experimental coefficients determining the viscoelastic properties of ferrofluid [$Pa \cdot s^2$],
η - dynamic viscosity coefficient [$Pa \cdot s$].

The materials coefficients α, β of lubricant liquid multiplied by the property rate of strain tensors take into account the additional stresses arising from the viscoelastic, non-Newtonian ferrofluid nature. In case of acceptance of material's coefficients α, β equal to zero can be obtained the classical Newtonian relationship between stress tensor and strain rate tensor.

Ferrofluid's dynamic viscosity depends mainly on the magnetic induction $\eta=\eta(B)$ and the material's coefficients α, β were taken as constants.

The amount of base of oil gap h_p depends on the relative eccentricity λ and nonparallelity of shaft and sleeve axis with an angle of γ see Fig. 1.

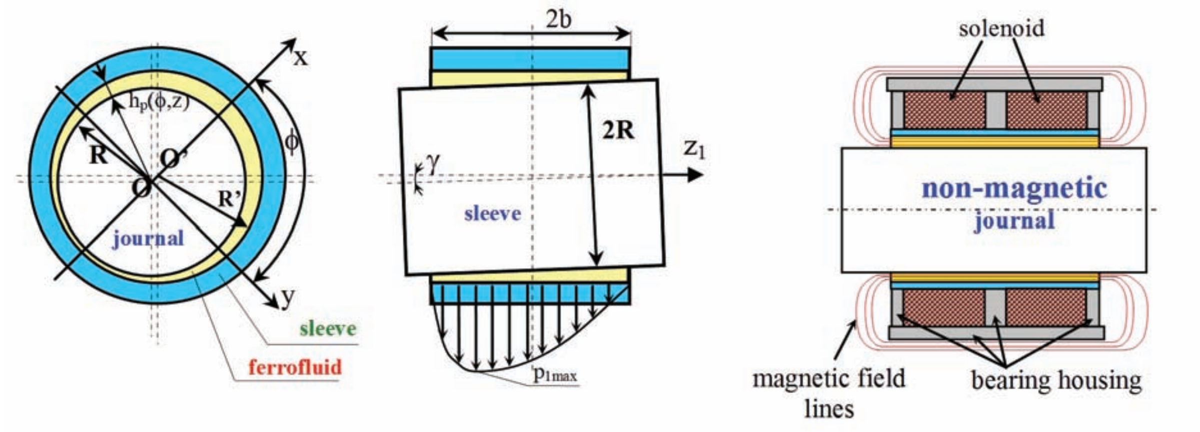

Fig. 1. The geometry of the oil gap of cross slide bearing in a magnetic field

The constitutive relations (6) between the coordinates of the stress tensor $\tau_{\phi\phi}$, τ_{rr}, τ_{zz}, $\tau_{\phi r}$, $\tau_{\phi z}$, τ_{rz} and the coordinates of the strain rate tensor are substituted for the equations of motion (1) - (2). So non-stationary units, as the forces of inertia in the equations of momentum shall be ignored. This kind of ignoration is justified in the slow and medium speed bearings. The whole set of equations of motion for classical isothermal steady flow of lubricating oil can be obtained in this procedure.

To estimate the magnitude of units in the system of equations and ignore small units of a higher order the making dimensionless and estimating the magnitude of units in the system of equations of conservation of momentum and continuity of the stream has been done. For this the following dimensional and dimensionless marks and numbers have been assumed [3, 6, 7]:

$$r=R(1+\psi r_1),\ z=bz_1,\ h_p=h_{p1}\cdot\varepsilon,\ p=p_o p_1,\ v_\phi=Uv_1,\ v_r=U\psi v_2,\ v_z=\frac{U}{L_1}v_3,\ p_o\equiv\frac{RU\eta_o}{\varepsilon^2},\ L_1\equiv\frac{b}{R},$$

$$\eta=\eta_o\eta_{1B},\ \rho=\rho_o\rho_1,\ \alpha=\alpha_o\alpha_1,\ \beta=\beta_o\beta_1,\ H_\phi=H_oH_1,\ H_r=H_oH_2,\ H_z=H_oH_3,\ N\ =N_oN_1,\ N_r=N_oN_2,$$

$$N_z=N_oN_3,\ \psi\equiv\frac{\varepsilon}{R}\cong 10^{-3},\ \eta_{1B}(\phi,z)=e^{\delta_B\cdot B_o\cdot B_1}=e^{\delta_{B1}B_1},\ R_f\equiv\frac{\mu_oN_oH_o}{p_o},\ De_\alpha\equiv\frac{\alpha_oU}{\eta_oR},$$

$$De_\beta\equiv\frac{\beta_oU}{\eta_oR}, \tag{11}$$

Dimensionless height of the oil gap changing in the circumferential direction and longitudinal direction is as follows:

$$hp1 = [1 + \lambda \cdot \cos\phi + a\gamma \cdot z1 \cdot \cos(\phi)], \quad a_{\gamma} = \frac{L_1}{\psi} \tan(\gamma), \tag{12}$$

where:

De_{α}, De_{β}, - Deborah's numbers as small dimensionless parameters,
H_o - dimensional value of magnetic field strength vector $[A \cdot m^{-1}]$,
H_1, H_2, H_3 - dimensionless components of the magnetic field strength vector,
H , H_r, H_z - components of the magnetic field strength vector $[A \cdot m^{-1}]$,
L_1 - dimensionless bearing's length,
N_o - dimensional value of magnetization vector $[A \cdot m^{-1}]$,
N_1, N_2, N_3 - dimensionless components of the magnetization vector,
N_{ϕ}, N_r, N_z - dimensional components of the magnetization vector $[A \cdot m^{-1}]$,
R - radius of the journal [m],
R' - radius of the sleeve [m],
R_f - dimensionless magnetic pressure number,
$U=\omega \cdot R$ - dimensional value of peripheral speed $[m \cdot s^{-1}]$,
a_{γ} - dimensionless skew factor,
2b - bearing's length [m],
h_{p1} - basic dimensionless height of the oil gap depends on the relative eccentricity and skew,
p_o - dimensional value of characteristic pressure [Pa],
p_1 - dimensionless value of hydrodynamic pressure,
r_1 - dimensionless radial coordinate,
z_1 - dimensionless longitudinal coordinate,
α_o, β_o - dimensional value of material's coefficients taking into account the influence of non-Newtonian liquids properties $[Pa \cdot s^2]$,
α_1, β_1 - dimensionless coefficients of material taking into account the influence of non-Newtonian liquids properties,
δ_B - dimensional coefficient of viscosity changes on magnetic field's induction $[T^{-1}]$,
$\delta_{B1} = \delta_B \cdot B$ - dimensionless coefficient of viscosity changes on magnetic field's induction B,
γ - the angle between the axis of the shaft and the axis of the sleeve commonly called skew angle,
$\varepsilon = R' - R$ - radial clearance [m],
η_{1B} - dimensionless value of dynamic viscosity dependent on magnetic induction,
η_o - dimensional value of dynamic viscosity for $T=T_o$; $p=p_{at}$; $B=0$ $[Pa \cdot s]$,
$\lambda = OO'/\varepsilon$ - relative eccentricity,
ρ_o - dimensional value of lubricant's density $[kg \cdot m^{-3}]$,
ρ_1 - dimensionless value of lubricant's density,
ψ - dimensionless value of the relative radial clearance,
ω - angular speed of journal bearings $[s^{-1}]$.

Dependencies (11) - (12) have been substituted into the equations of conservation of momentum and continuity of the stream. In this way, the system of equations in dimensionless form has been obtained in which one so the units with order of elementary magnitude are visible as units of the negligible order like the relative radial clearance $\psi \approx 10^{-3}$. Omitting units of the order of the relative radial clearance that means about a thousand times smaller than the value of other units, a new simplified set of equations has been obtained .

For further analysis of the basic equations it has been assumed that the dimensionless density $\rho 1=1$ of lubricant is constant and not depend on temperature and pressure [6, 7].

In order to solve the problem of hydrodynamic lubrication, which determine the size of the functions sought, such as: components of velocity, hydrodynamic pressure, load-bearing forces and friction forces the classical method of small parameter has been used. This method uncouples of nonlinear partial differential equations, forming two linear equations. The first set of equations allows determining the flow parameters for the classical isothermal, Newtonian lubrication with the magnetic field effects on the change in ferrofluid viscosity. The second one allows determining the so-called velocity components corrections and hydrodynamic pressure follow from the consideration of non-Newtonian properties.

Integrating twice after the radial variable corresponding momentum equation and applying boundary conditions circumferential and longitudinal component of the velocity vector have been obtained. Using the continuity equation and the previously evaluated components: the longitudinal and circumferential and after integrating equation and imposing the appropriate boundary conditions we obtain the radial component of velocity and Reynolds-type equation which has the form [3]:

$$\frac{\partial}{\partial\phi}\left[\frac{h_{p1}^{3}}{\eta_{1B}}\left(\frac{\partial p_1^{(0)}}{\partial\phi}-M_1\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_{p1}^{3}}{\eta_{1B}}\left(\frac{\partial p_1^{(0)}}{\partial z_1}-M_3\right)\right]=6\frac{\partial h_{p1}}{\partial\phi}, \tag{13}$$

where:

$$M_1=R_f\chi\left[H_1\frac{\partial H_1}{\partial\phi}+\frac{1}{L_1}H_3\frac{\partial H_1}{\partial z_1}\right],\ M_3=R_fL_1\chi\left(H_1\frac{\partial H_3}{\partial\phi}+\frac{1}{L_1}H_3\frac{\partial H_3}{\partial z_1}\right).$$

In the case of cross sliding bearing lubricated by ferromagnetic fluid with viscoelastic properties, the dimensional value of the lift force of bearing C is determined from the common known formula [3]:

$$C=C_1^{(0)}\cdot bR\eta_o\omega/\psi^2 . \tag{14}$$

Dimensionless value of the lift force $C_1^{(0)}$ of cross sliding bearing lubricated by ferromagnetic fluid with viscoelastic properties is calculated from the relation [3]:

$$C_1^{(0)}=\sqrt{\left(\int_{-1}^{+1}\left(\int_{0}^{\varphi_k}p_1^{(0)}\cos\gamma\sin\varphi\,d\varphi\right)dz_1\right)^2+\left(\int_{-1}^{+1}\left(\int_{0}^{\varphi_k}p_1^{(0)}\cos\gamma\cos\varphi\,d\varphi\right)dz_1\right)^2}, \tag{15}$$

where the symbol γ is the misalignment angle.

3. Numerical calculations

Numerical calculations of the pressure distributions and load carrying capacities are performed in Mathcad 14 Professional Program by virtue of the equation (13) (15) by means of the finite difference method (see Fig. 2). On the ground of such pressure distributions are calculated the load carrying capacities (see Fig. 3).

The numerical calculations of hydrodynamic pressure distribution and lift force have been performed for the relative eccentricity: λ=0.1; λ=0.2; λ=0.3; λ=0.4; λ=0.5; λ=0.6; λ=0.7; λ=0.8; λ=0,9 and the dimensionless length of bearing L1=1 at four concentrations of magnetic particles of ferrofluid: 0% (classic lubricating oil), 2%, 4% and 8%.

The components of the magnetic field strength have been determined by analytical-numerical solution of Maxwell's equations [4].

For all calculations, the following dimensional and dimensionless parameters have been assumed: angle of misalignment γ=0.000°; magnetic susceptibility corresponding to different concentrations of magnetic particles χ=2.0, χ=2.5, χ=3.0; the number of magnetic pressure Rf=0.5; dimensionless coefficient describing the effect of magnetic induction on the dynamic viscosity suitable for different concentrations of magnetic particles δB1 = 0.15; δB1 = 0.2, δB1 = 0.25. For determining the distribution of hydrodynamic pressure boundary conditions of Reynolds have been adopted.

Some hydrodynamic pressure distributions and load carrying capacities are shown in Fig. 2 and 3.

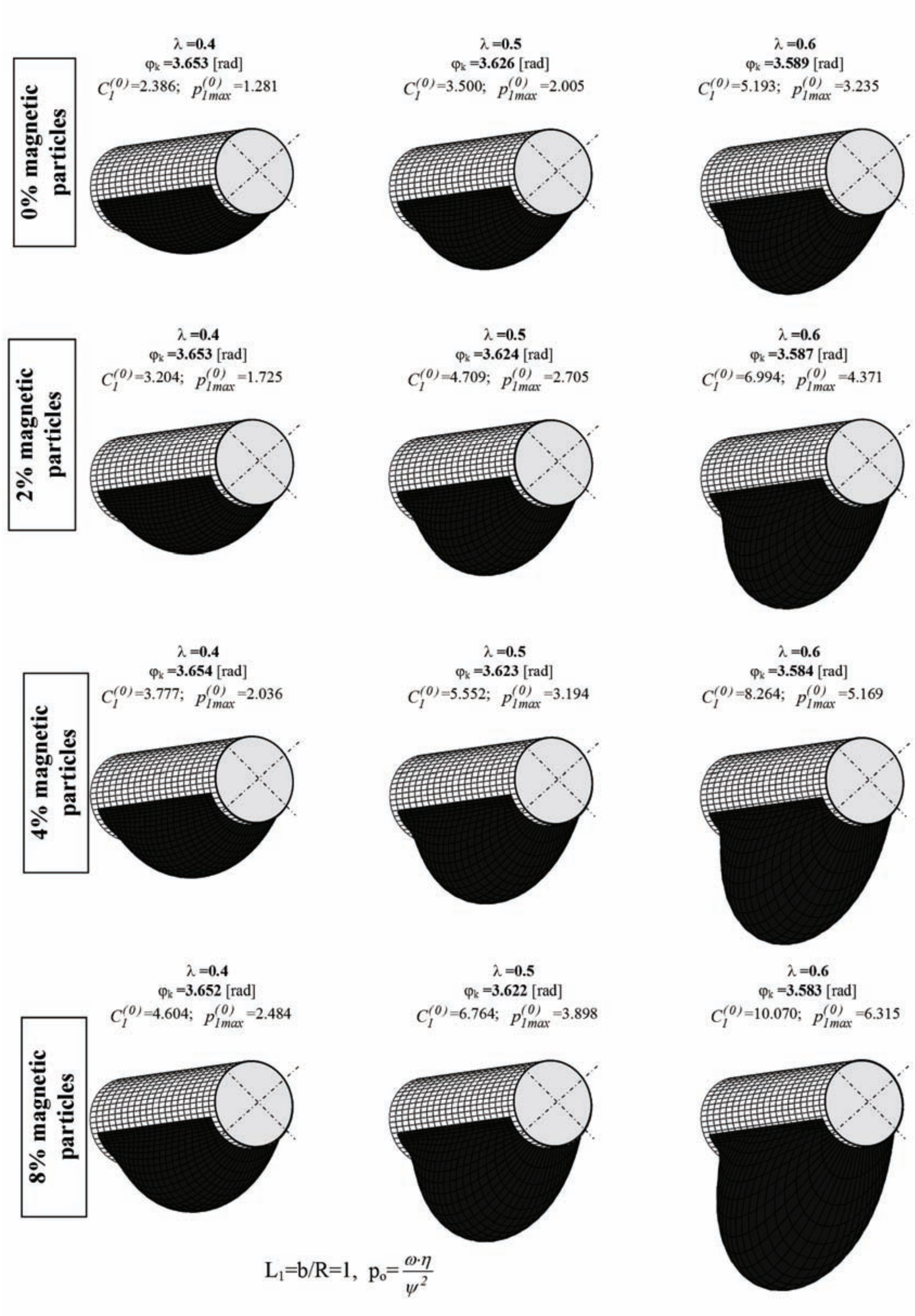

Fig. 2. The dimensionless pressure distributions $p_1^{(0)}$ *in cylindrical sliding journal bearings*

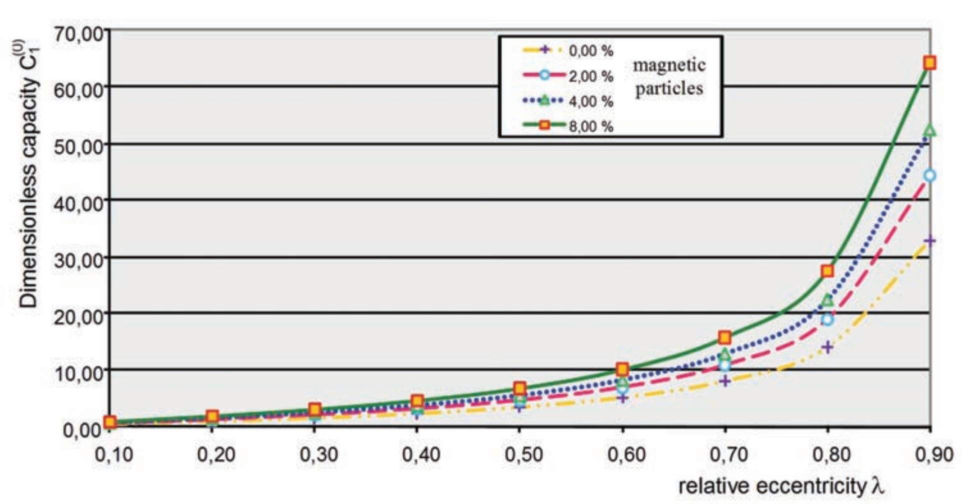

Fig. 3. The dimensionless load carrying capacities $C_1^{(0)}$ in cylindrical sliding journal bearings

4. Conclusion

Increase in the number of magnetic particles causes an increase in hydrodynamic pressure and lift force with the existence of the same value of external magnetic field. This increase is in the range: 30-95% according the lubricating oil without the magnetic particles. As the relative eccentricity is also increase in lift force but to a lesser extent - from 1-5%.

It should be clear that the quoted values are the result of computer simulation. The actual value of changes of hydrodynamic pressure and lift force will depend on the type of magnetic particles, the type of base fluid, the concentration of magnetic particles, the value of an external magnetic field, temperature and value of ferrofluid's hydrodynamic pressure which depends inter alia on the load bearing, rotational speed, radial clearance, and the geometric dimensions of the bearing.

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