PARAMETRIC MODELING APPLIED TO TURBINE BLADE OPTIMIZATION FOR THE REQUIREMENTS OF NATURAL VIBRATIONS ANALYSIS

STANISŁAW KACHEL, ADAM KOZAKIEWICZ
Military University of Technology

Abstract

In the article methodology for the construction of parametric model for natural vibration analysis of the turbine aircraft engine components was presented. The process of modeling in a systemic approach in application to turbine blade optimization was discussed. Peculiarities of the design of turbine aircraft engine and its aggregates were described. Algorithms of point selection out of the measurement data in order to create a pattern was formulated. In the paper a discuss is conducted on selection of the curves for parametric modeling including inputs to the process of surface mapping optimization based on reverse engineering technique. The process of geometry mapping begging from precise measurements, data identification, curve verification and ending with creation of a solid of modeled object is shown. This paper contains assumptions of algorithms of aircraft structure elements modeling developed and used by the authors.

Keywords: turbine jet engines, turbine, reverse engineering methods, profile curves optimization, cross-sections modeling, geometric design tools.

INTRODUCTION

Construction of the model in order to design new or reconstruct existing turbine blade is associated with the choice of multiple parameters, which relative to each other may be related by criteria and goal functions. Among many design methods most frequently encountered and effective way is hierarchical design, which steps are show in Figure 1. This multi-step process can be divided into three levels of design.

The first level is associated with the choice of engine type as a function of the tasks performed by the aircraft. The basic parameters that must be included at this stage are aircraft weight, velocity and altitude of flight and the number of engines. Simultaneously the conditions of optimizing work cycle of the engine must be formulated in order to determine the most comparable to the optimal design point together with parameters such as engine thrust $K$, temperature before the turbine $T_1^{*}$, compression of the compressor $\pi_{c}^{*}$, mass flow $\dot{m}$, etc [9].

The second level concerns the design and optimization of the components of the engines which include the air intake, compressor, combustion chamber, turbine and exhaust system, taking into account possible variants of design solutions. The process of turbine optimization
is to include size, weight, noise criteria, the criterion of reducing the temperature before the turbine, the turbine cooling criterion and a criterion for engine design time. On this basis charts of all of the criterions can be created in order to obtain areas free of restrictions for the turbine, which will determine the value of relevant work parameters of this component (e.g., expansion), but also other parameters of turbine jet engine.

The third stage is related to the formation of engine variants and determination of its performance which include maximum thrust $K_{\text{max}}$, minimum specific fuel consumption $c_j_{\text{min}}$, minimum weight $m_j$ and minimum dimensions of the unit as well as certain parameters relevant from the operational point of view as service life, cleanliness of the engine (exhaust gas composition) or noise level. As a result of the analysis area characterizing the engine with its mass and dimensions is obtained.

The final step is related to the calculation of the characteristics which should answer the question concerning the selection of the engine variant and its components (including the turbine component).

![Diagram of the design process of the turbine engines](image)

**Fig. 1. Scheme of the design process of the turbine engines [1], where $K$-engine thrust, $c_j$-specific fuel consumption, $m$-mass flow, $m$-engine weight**

1. **SINGULARITIES OF THE TURBINE STRUCTURE**

The turbine component of turbine jet engine (Fig. 2.) consist of many interacting elements, with different functional purpose subject to external influences that affect the efficiency of the entire system. In order to properly design this system a theory and methodology of complex systems must be used. This allows to obtain maximum performance in the range of all parameters and at the same time enables workmanship and quality of chosen solutions estimation.
The process of turbine component design should include:
- the complexity of the flow channel, which is a spatial layout
- the need to minimize all types of losses
- optimization of feather’s shape should be considered spatial

Fig. 2. The turbine component of two-spool turbine jet engine

These types of turbines with their need for specific design solutions resulting from the type and extent of the loading which they are subject to are difficult to describe due to all the relationships between elements of the system (functional elements properties and their relationships) [12]. Therefore to describe all these relationships hierarchical equations are used, from general to specific equations. In the highest level of the system (generalization) turbine can be considered as a whole (as a component of turbine engine), where the working medium of defined parameters (temperature \( T^*_3 \), pressure \( p^*_3 \) and velocity \( c \)) flows into and where the internal energy of the exhaust stream is converted into power of the turbine component \( P_T \) [5]:

\[
P_T = f(\dot{m}; T^*_3; p^*_3)
\]

where:
- \( T^*_3 \) - the stagnation temperature of exhaust gas
- \( \dot{m} \) - mass flow of exhaust gas
- \( p^*_3 \) - expansion in the turbine

In the modern aircraft turbine engines due to the high value of compression in the compressors (recently up to \( \pi_s = 49 \)) high temperature of exhaust gas before the turbine is required and it reaches \( T^*_3 = 1700 \ldots 1800 \) K. This gives \( \pi^*_T \leq 4 \ldots 5 \) for a single turbine.

The use of such a solution has huge advantages connected to simplifying the construction, eduction of the mass and lower demand for air flow for cooling (up to 1.5 times less than the two-stage systems). In the case of single-stage turbines for \( \pi^*_T \leq 4 \) with an appropriate degree of reactivity degree the flow velocity at the exit of the nozzle rim is subsonic and of the rotor rim is supersonic. If \( \pi^*_T = 4 \ldots 5 \) then for any value of reactivity a supersonic velocity is obtained, what requires appropriate modeling of the flow channel. Additionally significant circumferential velocities of \( u_{sr} = 530 \ldots 550 \) m/s are obtained on the mean radius, giving a kinematic coefficient of the flow \( u/c_{iz} = 0,46 \ldots 0,48 \), which however causes the increase in losses.
The second level of the hierarchic system is the division into high- and low-pressure turbine together with all the functional constrains. Next divisions (decomposition of the system) lead to the separation of each element, including very important element which is the turbine rotor blade (Fig.3).

![Fig. 3. Examples of turbine blades from turbine jet engines with different cooling methods](image)

Fig. 3. Examples of turbine blades from turbine jet engines with different cooling methods

- a - the type of blade with passive cooling,
- b - blade type with convection cooling,
- c - the type of blade with convection cooling of boundary layer

System is designed from the highest to the lowest level. In the decomposition additional attention should be paid to minimizing the change in constrains between elements and to the properties of these elements. This allows to examine each subsystem as a separate object with-component, due to the load, which determines the durability of the whole component is the cascade of the nozzle and rotor rim blades. Due to these factors, much attention is paid to the design of turbine blades. Turbine blades in aircraft turbine engines are the elements loaded at most and therefore affecting durability, reliability and safety of the engine operation. Turbine blades (as well as compressor blades) belong to most numerous group of elements occurring in turbine engines. Number of these elements may reach the order of 3500 pieces (with nearly 500 blades on a single stage). Service life span of the blades of this component is quite large and it ranges from 500 hours (for multi-role military aircrafts) up to 20,000 hours (for engines used in airliners) [6]. Life-time of the blades depends on fixed loading (especially transient loading), which turbine component is subject to.

2. MATHEMATICAL MODEL OF THREE-DIMENSIONAL BLADE

The wording of the task of virtual model creation using reverse engineering method is distinguished by a large number of optimized variables, mostly of a combinatorial or combinatorial-cyclic form of imposed constrains, associated with geometric limits. Imposed restrictions
do not allow for a direct solution without division into related set of tasks solving various stages of modeling. The division of the general task and solving separated subtasks is based primarily on the specific characteristics of the optimized quality index of reconstructed object and relationships and present limitations of the initial task (output). An important factor for effective solution of this problem is the possibility of separating the stages of the virtual object creation, belonging to the most important class of mathematical tasks.

The mathematical equivalent of the variant selection in the process of virtual object modeling (CMM) is an optimization of quality index, which is based on the process of coordinate measuring. The task of the measurement optimization is to determine the characteristics, which would provide an adequate distribution of the distribution function $E(x)$ of the assumed process (A) of modeling, leading to a solution on the basis of measurements $\{y_x(t), y_x\}$, optimal from the viewpoint of minimizing the quality index $F$.

$$F(X, A, \{y_x\}, \{u_x(t)\}, E(x)) = \min F[X, A, \{y_x\}, \{u_x(t)\}, E(x)]$$  \hspace{1cm} (2)

where the minimalization of the functional is subject to the following conditions:

$$A \subset Y$$

$$E(x) \subset \{1, \ldots, n\}, \ n \geq 1$$  \hspace{1cm} (3)

Formulating a strategy that will cover physically realized process that fulfills the execution of measurement tasks defined in the set $X$ is as follows:

$$A = \{y_j\}, \ y_j \in Y_D(y_j), \ j = 1, \ldots, n$$  \hspace{1cm} (4)

If one type of mapped object is considered then the process comes down to the task: the description of the structural design (geometric) and the main characteristics of elements of the designed or reconstructed object. These assumptions in relation to a jet engine turbine blades can be divided into two stages. Modeling of the blade root taking into consideration geometrical strength criteria and modeling of the blade, where aerodynamic criteria should be included, which in this case is closely related to the geometric criterion of the $\gamma \in Y_x$ type.

Isolation of a specific task is conditioned by complex structure of the set of acceptable parameters $y, Y_D$ as well as significant computational effort of the procedure for determining parameters of reconstructed objects that satisfy the condition $\gamma \in Y_D$. The procedure for implementing particular task is reduced to the solution of nonlinear equations representing the quantitative properties of the searched object.

$$G_i(\eta) = 0, \ i = 1, \ldots, l$$  \hspace{1cm} (5)

$$H_j(\eta) \geq 0, \ j = 1, \ldots, r$$  \hspace{1cm} (6)

where the system of equations (5), (6) provides $l$ independent equations and $r$ inequalities against $k$ variables, $\eta$ that describe the basic constructional data and the properties of the object. The degree of difficulty and complexity of the task is to build system of equations and its immediate solution by computational methods [2,11].

Each system of equations (4) describes in $k$-dimensional space a $(k-l)$ dimensional subspace with $l$ equations of $(k-l)$ degrees of freedom. In inequality (6) acceptable areas of solutions (5)
are separated. For example any vector $\eta \in \mathcal{H}$ can be uniquely given by its $(k-l)$ components. Other elements are determined for equations (5) and (6).

The output set of variables $\eta = \{\eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(k)}\}$ is usually divided into $p=k-l$ independent variables $\mathbf{y} = \{y^{(1)}, y^{(2)}, \ldots, y^{(p)}\}$ and $l$ dependent variables $\mathbf{\omega} = \{\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(k)}\}$ determined from the system (6) according to the given value $y$. Independent variables are usually called design parameters. To significantly facilitate the resolution of this problem gives the possibility of using the GRIP language [7,8] for the Unigraphics system.

3. BLADE PROFILE GEOMETRY MODELING USING INTERPOLATION SPLINE FUNCTIONS

Interpolation tasks appear frequently in order to approximate solving of approximation tasks. The creation of the curve on the basis of CMM (Fig. 4) and its reflection in numerical representation to approximate the surface stretched on created curves, it is possible to define and solve the interpolation problem. To solve the task formulated in this manner, coordinates are recorded for a number of points on the curve or surface, and then based on these measurements an interpolation curve or surface is determined. If accuracy of the shape reproduced by found interpolation curve is too low, then it is natural to „thicken” the data, which means to give extra points, through which the curve is going to pass.

![Fig. 4. The points obtained from the database](image)

The shape of the interpolation curve is influenced by the class of the curves in which we are looking for solution of the problem. If the curve, which we are constructing, is polynomial and between given points shows waving, adding points (and enabling comparatively higher level) will result is a curve of much larger waving. Therefore, mentioned thickening of the data is permitted only when it is known that it will reduce the error of approximation. Interpolation spline functions (and curves) are often used, inter alia, because in many cases they provide a good solutions to approximation tasks, thickening of the interpolation nodes allows to reduce the error of approximation of given function (or curve). Knowing the estimate of second-order derivative of given function it is possible to calculate the number of nodes, which ensures a sufficiently small error in the application. Based on this assumptions the procedure for reading data files containing measured points in GRIP language was developed.
Second-order derivative of cubic spline function is a continuous function, which in an interval $[u_i, u_{i+1}]$ is a polynomial of first degree. Denoting $x_i = s(u_i) = f(u_i), y_i = s'(u_i), z_i = s''(u_i)$, where polynomials $p_{i,1}, p_i$ describe the function $s$ in the intervals respectively $[u_{i-1}, u_i], [u_i, u_{i+1}]$. By introducing a variable $v = t - u_p$ the derivatives of second-order polynomials $p_{i,1}$ and $p_i$ are solutions of the Lagrange interpolation tasks:

$$p'_{i-1}(t) = \frac{z_i - z_{i-1}}{h_{i-1}} v + z_i, \quad p''_{i}(t) = \frac{z_{i+1} - z_i}{h_i} v + z_i \tag{7}$$

Double integration of (7) allows to obtain polynomials $p_{i-1}$ and $p_i$. For this we choose the constants of integration so that $p_{i-1}(u_i) = p'_i(u_i) = y_i i p_{i-1}(u_i) = p_i(u_i) x_i$. As a result of the transformation is obtained:

$$p'_{i-1}(t) = \frac{1}{2} \frac{z_i - z_{i-1}}{h_{i-1}} v^2 + z_i v + y_i \quad p'_i(t) = \frac{1}{2} \frac{z_{i+1} - z_i}{h_i} v^2 + z_i v + y_i \tag{8}$$

$$p_{i-1}(t) = \frac{1}{6} \frac{z_i - z_{i-1}}{h_{i-1}} v^3 + \frac{1}{2} z_i v^2 + y_i v + x_i \quad p_i(t) = \frac{1}{6} \frac{z_{i+1} - z_i}{h_i} v^3 + \frac{1}{2} z_i v^2 + y_i v + x_i \tag{9}$$

Where value $p_{i-1}(u_{i-1})$ and $p_i(u_{i+1})$ are obtained by substituting respectively $v = -h_{i-1} i v = h_p$ and the result is:

$$x_{i-1} = -\frac{1}{6} (z_i - z_{i-1}) h_{i-1}^2 + \frac{1}{2} z_i h_{i-1}^2 - y_i h_{i-1} + x_i \tag{10}$$

$$x_{i+1} = \frac{1}{6} (z_{i+1} - z_i) h_i^2 + \frac{1}{2} z_i h_i^2 + y_i h_{i-1} + x_i \tag{11}$$

Basing on the equations (10) and (11) we can find $y_i$. Both expressions obtained in this way must be equal to:

$$\left| \frac{\frac{x_i - x_{i-1}}{h_{i-1}} + \frac{1}{3} z_i h_{i-1} + \frac{1}{6} z_{i-1} h_{i-1}}{h_{i-1}^2} \right| = y_i - \frac{x_{i+1} - x_i}{h_i} - \frac{1}{3} z_i h_i - \frac{1}{6} z_{i+1} h_i \tag{12}$$

After transformation the equation is obtained

$$\frac{h_{i-1}}{h_{i-1} + h_i} z_{i-1} + 2 z_i + \frac{h_i}{h_{i-1} + h_i} z_{i+1} = 6 f[u_{i-1}, u_i, u_{i+1}] \tag{13}$$

which must be satisfied for $i = 1, \ldots, N - 1$

$$\left| f(t) - s(t) \right| = \left| f(t) - p_i(t) \right| \leq \left| f(t) - x_i - \left( \frac{x_{i+1} - x_i}{h_i} - \frac{1}{6} (2 z_i + z_{i+1}) h_i \right) v - \frac{1}{2} z_i v^2 - \frac{1}{6} \frac{z_{i+1} - z_i}{h_i} v^3 \right| \leq$$

$$\left| f(t) - x_i - \frac{x_{i+1} - x_i}{h_i} v \right| + \frac{1}{6} (2 z_i + z_{i+1}) h_i v - \frac{1}{2} z_i v^2 - \frac{1}{6} \frac{z_{i+1} - z_i}{h_i} v^3 \tag{14}$$

The Lagrange interpolation task was recorded in the procedures of GRIP language developed for Unigraphics. This task has exactly one solution $p(t)$. 

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**Parametric Modeling Applied to Turbine Blade Optimization for the Need of Natural...** 53
The following is a result (Fig. 5) of usage of this method to determine \( p(t) \) based on the GRIP language.

\[
p(t) = \sum_{i=0}^{n} p_i \left( \prod_{j=0, \neq i}^{n} \frac{t-u_j}{u_i-u_j} \right)
\]

(15)

The solution of equations (13), supplemented by boundary conditions, for example \( z_0 = z_N = 0 \), gives an alternative way of constructing an interpolation function or third degree spline curve of \( C^2 \) class.

Fig. 5. Points received by the selection of the Lagrange interpolation coefficients method

Fig. 6. Algorithm for selecting the point \( p_i \) from measurement data to create a geometry model

The construction of a geometric model, which is a base to verify physical objects compared by measurement method is based on own algorithms [7] for the Unigraphics system. Figure 6 summarizes essential parameters used in procedures to automate the process of reconstructing the jet engine turbine blade.
The final result of the above procedure is shown on the blade element which is a multi-trapezoidal root of the blade (Fig. 7).

4. THE DESIGNING OF CURVES DESCRIPTION THE AERODYNAMIC PROFILE

Theoretical considerations

Theoretical considerations should being with the question what is the minimum degree of the curve to ensure proper distribution of the pressure on the profile (Fig. 8) described by assumed function, which equation can be used to reconstruct the profile of the blade.

\[ c_p = 2 \frac{P_i - P_1}{\rho V_1^2} \]

The answer to the question may be formulated by the equations describing the inviscid flow against searched curve, considering the flow along the current line, including the conditions of tangential velocity of the flow in each point of the profile defined by the equation of the curve and conditions along normal to the profile in the point.
By design, the surface of the thin profile is formed along the flow, therefore the equations (16) and (17) can be used in the vicinity of the considered curve. It is obvious that the line defining the outline of the profile (Fig. 9), which is a solution to formulated task, depends on global parameters such as the undisturbed flow angle and the angle of the stream departure from given profile.

![Image of aerodynamic profile](image)

**Fig. 9. Construction of the aerodynamic profile including design angle of attack and trailing angle**

It should be noted that the turbulence that occurs between the input parameters have only a local effect on the flow field. The basic assumption is that change in the line of the current caused by the curvature is of a small value. In other words, the main hypothesis is as follows: the effect of modifications to the thin profile of the transition region is negligible.

Referring to the aforementioned hypothesis [4] the modification of thin profiles can be easily done by proper pressure distribution adjustment on the given profile.

**B- spline**

Application of spline functions in computer-aided systems made it possible to use parametric approach to the modeling of the virtual objects.

B-spline curve is zero in all subintervals of the parameterization except m+1. Such curves can be defined recursively as follows:

$$N_{i,0}(x) = \begin{cases} 
1 & x_i \leq x \leq x_{i+1} \\
0 & dla \ pozostalych \ x 
\end{cases}$$

(18)

B-spline degree m in the intervals $[x_i, x_{i+m+1}]$ is defined as:
Taking into account equation (18) and (19) an explicit form of the B-spline curve of any degree can be found, which is directly used in the algorithms for plotting curves in CAD systems. For B-spline linear value:

\[
N_{i,1}(x) = \begin{cases} 
\frac{x-x_i}{x_{i+1} - x_i} & x_i \leq x \leq x_{i+1} \\
\frac{x_{i+2} - x}{x_{i+2} - x_{i+1}} & x_{i+1} \leq x \leq x_{i+2} 
\end{cases}
\]  

(20)

For B-spline quadratic value is:

\[
N_{i,2}(x) = \begin{cases} 
\frac{(x-x_i)^2}{(x_{i+2} - x_i)(x_{i+1} - x_i)} & x_i \leq x \leq x_{i+1} \\
\frac{(x_{i+3} - x)(x-x_i)}{(x_{i+2} - x_i)(x_{i+1} - x_i)} & x_{i+1} \leq x \leq x_{i+2} \\
\frac{(x_{i+3} - x)^2}{(x_{i+3} - x_{i+1})(x_{i+3} - x_{i+2})} & x_{i+2} \leq x \leq x_{i+3} 
\end{cases}
\]

(20a)  

(20b)  

(20c)

Using presented approach the shape of an aerodynamic profile can be easily described and it can be used in the construction of aircraft engines turbine blades.

To carry out the optimization of selection of points for curve that reproduces the aerodynamic profile a program in GRIP language was developed for UNIGRAPHICS.

As a result of the program implementation a set of points that reproduce searched curvature of the profiles on well-defined heights of the blade is obtained. Which in turn enabled the construction of blade model (Fig. 10).

Fig. 10. Result of searching for points reproducing the curves of blade profiles.
5. NATURAL FREQUENCY OF BLADE

Calculation of vibration of engine components, including turbine blades, is an essential process in the turbine jet engine design (and already exploited) in order to protect the system against the possibility of structural resonance occurrence. There are many reasons of blade vibrations. Turbine blade is a complex spatial structure, which is loaded with centrifugal force, pressure of the gas flow and pressure of the cooling stream.

![Image of turbine blade vibration frequency]

Fig. 11. The first four characters of turbine blade vibration frequency. a - the first form of vibration, b - the second form of vibration, c - the third form of vibration, d - the fourth form of vibration

FINAL REMARKS AND CONCLUSIONS

The operation pf aircraft and its engines, for which there is no tehnical documentation (aircraft purchased from another country), raises many questions and problems among others related to the engine. It is true that the manufacturer determines the operating conditions but does not reveal the secrets that can be a source of new technologies or new technologies for the industry of the buyer country, This truth was confirmed when buying airplane Su-22 and MiG-29 from former Soviet Union and most recent purchase of U.S. F-16 aircraft.

There is thus a need to conduct in Poland own engine analysis, which are used in aircrafts operating in Polish Air Force. Hence, one of the most important issues is the need, regardless of manufacturer, for finding characteristics of rotor assemblies of the aircraft engines. This knowledge is very useful in the process of aircraft ongoing operation and involves the need to ensure the safety of the flight.

This area of analysis and works contains also design and reconstruction of engine components as well as aircrafts in the CAD/CAM/CAE systems. Principal parametric model is created based on characteristic sizes (points), which are geometric boundary conditions of modeled elements, which often leads to forced changes in the approach to the design process at the stage of description of the model in the integrated CAD/CAM/CAE system.
The process of elimination of measurement errors affecting the determination of basic parameters of the object based on the algorithm of selection of the point [2] optimization causes the decrease in time of obtaining the virtual model useful to geometry analysis of the object (smooth surfaces). Key benefits of the proposed algorithm of creating parametric model described with procedures of GRIP language are:

- elimination of inconvenient geometry corrections that prolong the time to create geometric model
- reduction of the number of variables in the process of determining the characteristic values
- possibility to determine the rules of inference to retain the intermediate parameters.

Establishing the rules of reconstruction and modification allows to change geometry without changing parameters imposed by the constructor.

The components of geometric model form the basis to create elements of the aircraft structure that are necessary to conduct geometric and mass, strength and technology analysis.

BIBLIOGRAPHY

METODYKA BUDOWY MODELU PARAMETRICZNEGO NA POTRZEBY ANALIZY DRGAŃ WŁASNYCH ELEMENTÓW LOTNICZEGO SILNIKA TURBINOWEGO

Streszczenie
W artykule zaprezentowano metodykę budowy modelu parametrycznego na potrzeby analizy drgań własnych elementów lotniczego silnika turbinowego. Omówiono proces modelowania w ujęciu systemowym w zastosowaniu do procesu optymalizacji łopatki turbiny. Przedstawiono osobliwości projektowania lotniczego silnika turbinowego i jego zespołów. Opracowano algorytmy wyboru punktów z danych pomiarowych do utworzenia wzorca. Przeprowadzono dyskusję doboru krzywych do parametrycznego modelowania z uwzględnieniem wejść do procesu optymalizacji odwzorowania powierzchni bazując na technice inżynierii odwrotnej. Przedstawiono proces odwzorowania geometrii od etapu wykonania precyzyjnych pomiarów, identyfikacji danych, weryfikacji krzywych, aż do utworzenia bryły modelowanego obiektu. W pracy zawarto założenia opracowanych przez autorów i zastosowanych algorytmów modelowania elementów struktur lotniczych.