1. ACTIVE SYSTEMS

1.1. Active systems - introductory remarks

In different subject areas we encounter a need of a qualitative or a quantitative level of evaluation, forecast, and description of man- subject participation.

Practically each kind of activity can be represented as a functioning of a certain system that is more or less individualized, in center of which a subject – an active element of the system stands. Its role is varied and will be gradually analyzed and refined further on. *The system, in center of which the subject is situated, that to a considerable extent determines the system functioning, is called an active in contrast to passive system, for example, purely technical, natural, which doesn't include active elements?*

The said in a strict sense is not the definition of an active system yet. This definition will be formed subsequently. The revelation between objective and subjective factors, which characterize system and establishment of the boundary between these features, is one of primary tasks of active systems studying.

In the work [64] the concept of „active system" was discussed in *„the first approximation"*. It was assumed that, by main property, the distinctive special feature of the active system is its ability *to generate its own problems*. Functioning of an active system in this meaning is considered as the permanent activity, directed to a solution of its own problems. It is assumed that each active system at each moment is situated in a certain problem- *resource situation*. Any alternation of situation is the objects of „situation dynamics".

By the *subject* we understand both individual and certain group of individuals of those connected with of general problems and by consolidated resources. In the latter case active system can be subjected to decomposition on subsystems which are in „vertical" (hierarchical), or „horizontal" relations. Such systems have their own internal structure. Structural changes are caused both by a change in external factors and „spontaneous" changes, which also could be considered a distinguishing feature of active systems a designated by term „self-organizing"? In this connection active system can be considered as an object of synergetic.

We have to work out such methods of analysis and synthesis which would consider in an explicit form subjective factors, connect with an activity of a „subject" – „decision making person" accomplishing the activity, directed to realization of the accepted solutions. It's namely such a sense we have in mind speaking about „subjective analysis".

Methods, being the subject of this discussion, have to enable carrying out analysis, processing of statistical data with a obviously designating purpose and in the specific terms, the identification of such systems quantitative models, their synthesis, prediction and control.

One of tools of description and study of active systems is *the problem- resource method* [47, 64, and 69], consonant with the sufficiently detailed elaborated utility *theory* [27, 38, 53, 61, 113, 118, 119, 125, 163, 165, 171, 184, 194, 197, 198, and 199]. The analogy of simplest problem in the utility theorem [149] is a transitive binary relation of preference ρ determined on the set alternatives S_σ, quantitative measure of which is a utility function.

The utility theory arises in works of the 18th Century economists. Later as a quantitative measure of preferences a utility function begins to use. By important stake in a development of the quantitative theory of the utility became the work of Neumann and Morgenstern [125]. Subsequent development stages of theory are described in monograph [149]. We will refer to this work later on.

Subsequently productive proved to be a synthesis of the utility theory with the new direction of studies, which today is designated as „synergetic".

By the founders of synergetics are considered H. Haken and I. Prigogine [128, 132, 133, 151, 152, and 153]. Term „synergetics" belongs to H. Haken a specialist in region of quantum mechanics, theory of coherent emission, non-equilibrium phase conversion.

Studies in the field of physics of the non-equilibrium self-organizing systems led to understanding of the fact that effects of self-organizing on the macroscopic level can have an actuality in wider spectrum of realization, and the synergetics as scientific discipline go far out of the scope of physics. Effects of self-organizing in biology, economy [9, 11, 12, 22, 56, 84, 86, 99, 100, by 101, 121, 126, 134, 155], in social systems and structures were investigated. As an example from the range of biology the Belousov-Zabotinsky reaction, stage of development of a fungus and other, more complex phenomena could be mentioned. Ideas of synergetics penetrated in psychology [127, 153] and such extremely complicated region as processes of development and formation of cultures, political and civilization processes - generation, development and the loss of civilizations [30, 41, 55, 60, 84, 86, 121, 122, 133].

Synergetics cannot pretend to be on exceptional rights of description and explanation of objects mentioned above. However, it is an effective tool of studies, including quantitative, badly formalized processes and phenomena.

1.2. Notions and criteria of the problem- resource analysis

In this division basic concepts and categories of the problem - resource analysis are discussed. At first we carry out concepts of *„active system"*, *„subject"*, *„problem"*, *„purpose"*, *„resources"* and some others.

We will also discuss a connection between problem - resource analysis and other known theories and methods, including the theory of binary relations [149].

In the previous division we began a discussion about a concept of an *„active system"*. Let's continue this consideration. It is obvious that any active system is placed in a certain enviroment and it is assumed that there is a possibility to individualize system, i.e., to indicate its boundaries in hyperspace of factors or characteristics which are selected for describing of the system.

The studied system interacts with *„an environment"* and in this meaning it is *open*. Environment can be natural, or one including other active systems. It is understandable that in the number of characteristic of the active system a description of methods and „channels" of interaction with the environment should be included.

As it was already said, *subject* is a central component of an active system. Therefore it is natural to call analysis of active system subjective analysis. The active system is grouped „around" the subject similarly to the components of a living cell that are grouped around its nucleus. If there is no subject - there is no active system. Technogenic or natural systems are not an object of this theory. A presence *of subject* assumes a presence *of the object* of his activity. Resources (including - other active systems) are objects of activity in the majority of the cases.

Generally, functioning of the active system with a known imagination can be interpreted as conversion and displacement (translation) of resources. The conditionality of this interpretation we will discuss below.

Because of this we are forced to study category „resources", classification of resources, processes of their conversion and translation. Resources are *the object* in active system, which the activity *of subject* is inverted to.

First of all the system is characterized by a collection of possible states.

Let's designate the certain state as σ_i. Then $\sigma_i \in S_\sigma$. The state σ_i is understood as the complex characteristic - „vector", comprised with particular characteristics (qualitative or quantitative). *Possible* will be count such a state upon transfer in which system remains *„itself"* (it does not lose its individuality). It means that, first of all, *the subject* remains *the same*. Secondly the certain collection of technologies of resources conversion and translation, kept constant. Also, the certain collection of basic links with other systems remains.

It is clear that this definition is not strict and exhaustive, but an absolute strictness - „the closure" of definition here from the point of view is impossible of the author and even it can be harmful brake in the development of the theory.

What, nevertheless, does the expression: „System does remain still itself" or „subject does remain the same" indicate?

Within the framework of the subjective analysis, object of which are active systems and their functioning, we must try in any manner to define concretely the concept *of the active system*, to indicate such a „*constant*" - *invariant* and, in the same time, essential, which makes it possible to operate with this concept, to build quantitative and qualitative models, to speak about interaction of systems.

It is intuitively clear that the active system, as for as it is substantially connected with its nucleus - subject (man), group of subjects, it must reflected in its „formalized" determination the subject basic properties. Let's note that first of all we are, of course, interested in the mental properties and, to a considerably smaller degree - physiology.

Physiology can be essential only in the sense that different periods in man's life are characterized by different collections of possible states, collections of alternatives and preferences of alternatives, the fact that the physical life of subject is final and therefore, „the life" of the active system, connected with individual is final too. With each individual elementary, individual active system is connected. Each social, which consists of *M* individuals, generates *M* elementary active systems. However, within the limits of each social occur a dynamic processes of composition and decomposition of active systems, a formation and disintegration of coalitions (unions), consolidation and deconsolidation of resources, aggregation and a disaggregation of individual and groups of alternatives, a change in the type of preference relations.

So let's accept the following compromise definition, which is at the same time the assumption: active system remains itself, is individualized as long as there is „a subject" of system (physically or legally), who has the non-empty set of possible alternatives S_σ, who establishes on this set (strictly speaking – on the product $S_\sigma \, x \, S_\sigma$) preference relation $\rho : \precsim$ and have available resources (passive and active). Important from the point of view of this definition is concept of „active resources", since operating by passive resources each time requires the use (expenditures) of active resources. Thus, the exhaustion of active resources is equivalent to the curtailment of the functioning of active system. It is necessary to assume that the structure and the scales of alternatives set are connected in a defined manner with the presence of active resources.

„Tightening" of alternatives set S_σ to an empty set or an appearance of an indistinguishability of alternatives from the given set, if this state is steady one, indicates the curtailment of „external" functioning of the active system. It becomes indistinguishable from without. This state can be treated as „*entropy*" death (not physical), while striving for to zero of active resources is equivalent to „physical *death*". More accurately, apparently *physical death* is such state, when expenditures of passive resources, necessary for a renewal of the active resources of subject become infinite. This not medical (not physiological) determination of death is clearly understood. This is the definition, which makes it possible to individualize the active system within the framework of the subjective analysis.

The given reasoning's are „leading" and debatable. In this stage they give a certain right to speak about the active system as about the chosen, individualized and identified subject of a study. In this case by identifiability we imply a possibility to determine „*the boundary*" of this active system, which separates it from other active systems in „*an environment*" (Fig. 1.1).

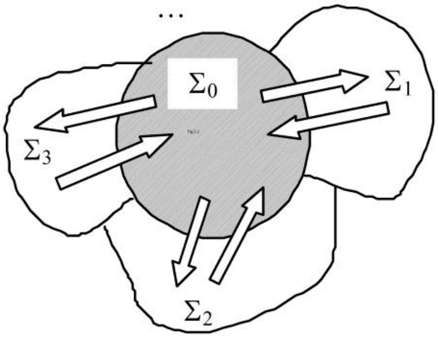


Figure 1.1

It is understandable that, first of all we can demarcate resources, which this system manages from resources belonged to the other systems. Then we should describe connections, existing between this system and an environment. It seems that these connections are realized in the form of mutual transfer of resources (material, energy, information). In a character of information resources finances, directives, advices, recommendations, any economic, political, technical, military and other information can come out. With the transfer of resources they change „owner". That mental boundary on which this occurs can be treated as the boundary of active system. Is it possible to transmit active resources? It is seemed that „*active resources*" - this is such a specific form of resources, which cannot be transmitted, from one individual to another. These assertions can be considered with the determination of what are „*active resources*". Thus „*active resources*" are not transferred between the individuals, but with an interaction of individuals in the group „cumulative effect" of a growth of active resources, has place due to an intensive information exchange. This can occur in the process of instruction, in the command sport games and so forth.

In social or in groups the processes of self-organizing which particularly are object of synergists training occur as a result of interaction of individual „active systems".

In the work [55] as the active system is considered those capable to export entropy (in our case subjective entropy). As it will be seen in future active systems could decrease it's entropy by itself.

Let now certain part of states of set S_σ is studying by the subject from the point of view of their priority for a realization on next step. A collection of such comparable states we will designate through $S_a \subset S_\sigma$, and the corresponding states through σ_i:

$$\sigma_i \in S_a \subseteq S_\sigma.$$

It is convenient to call states σ_i in this case alternatives. The selection of set S_a is a result of subjective analysis. Thus, set S_a is the important characteristic of subject. In the concept *of alternative* can be included not only states from S_σ, but also the admissible strategies of reaching these states. Let U_σ set admissible strategies of passages in the states $\sigma_i \in S_\sigma$. In this case it is, of course, assumed that the system is in a certain initial state σ_0. The permissible set is always locked and determined each time by resource limitations.

Let $U_a \subseteq U_\sigma$ be set of strategies of those studying by subject. Let's designate the Cartesian product of sets S_a and U_a trough W_a:

$$W_a = S_a \times U_a.$$

We will write $\sigma_i \in W_a$ bearing in mind that σ_i can be both a desired state and a strategy of reaching a desired state.

In *the utility theory* it is assumed that subject forms on the set W_σ his preferences. *The utility function*, which is a quantitative measure of preferences in the majority of cases, is not calibrated and serves for determining a relative value of alternatives ,σ_i.

Subsequently for simplification of designations we will instead W_σ (or W_a) also write S_σ (or S_a) specifying each time what is an intention: state or strategy.

We must consider the important circumstance that occurs regardless of the fact, which we do examine as a system a small particular enterprise or a history of the development of the entire peoples: *any active system has a limited „sizes" in space and the final „lifetime"*.

Hence it follows in particular, that functions of preference, a distribution of preferences on S_a as well as set alternatives S_a vary with time not only as a result of a change in external (exogenous) conditions and train of selected problem solution, but also „it is spontaneous", i.e., it vary as a result the action of the original properties of psyche and physiological processes, in other words - by an action of endogenous factors.

1.2.1. Problem

Problem- resource analysis is first of all based on a concept „*of problem*".

Problem is understood as a realized nonconformity between an existing state of active system and its desired state. In other words, problem is the realized desire of subject, or the realized preference - consequence of the desire.

In this definition as minimum two states present σ_e is exist, and σ_d is desired, and also „*the carrier*" of this „realized desire" - *subject*. Therefore the problem does not exist apart from its carrier - subject. It is assumed that *the desires* of subject are distributed on a certain set S_a (or perhaps W_a). Depending on the fact what „physically" the states σ_i are set S_a can be countable (including - finite), or continuum.

In the indicated sense the simplest problem can be interpreted as an ordered pair of the symbols:

$$P\colon \sigma_e \langle \sigma_d, \tag{1.1}$$

or

$$P\colon \sigma_e \langle \forall\sigma_d \in S_d \subset S_a.$$

In the second case each state of the subset S_d of set S_a is better (preferred) then initial state σ_e (existing state). It is possible to say, that each state from a certain subset $S_1 \subset S_a$ is better than each state from $S_2 \subset S_a$:

$$\forall\sigma_i \in S_1 \langle \forall\sigma_j \in S_2 \; (S_1 \subset S_a \text{ and } S_2 \subset S_a). \tag{1.2}$$

A problem is attached to *the existing* state σ_e, while set of problems is attached to the set $S_a \subset S_\sigma$. Selection S_a from S_σ is a subject's prerogative and therefore, it is subjective.

The set of problems, assigned on S_a x S_a differ from the set of alternatives S_a regarding the fact that the first it is always connected with the existing (achieved at the given moment) state $\sigma_e \in S_a$, $\sigma_0 \,\overline{\in}\, S_a$. Preference relation can be strict $\rho : \langle$, or lax $\rho : \underset{\sim}{\langle}$, allowing equivalence. „Problem" we will consider a strict relation. The problem can represent desire to preserve existing state σ_e. This corresponds to the case, when from the point of view of subject all states $\sigma_i \in S_a$ $(i \neq e)$ are less preferable than the state σ_e, subject himself is found in at the given moment.

Number of binary relations ρ in S_a are equal N^N. In the general case preference relation can be realized in σ - algebra A_σ of subsets S_a:

$$A\rho B \Leftrightarrow A \underset{\sim}{\rangle} B, \quad A, B \in A_\sigma . \tag{1.3}$$

As it has already been mentioned, the simplest problem can be interpreted according to the symbolism accepted in the theory of binary relations as a binary relation ρ:

$$P: \sigma_e \langle \sigma_d \Leftrightarrow \sigma_d \,\rho\, \sigma_e.$$

However, there are special features in a formulation of the basic concepts that make it possible to consider problem- resource analysis as an original theory that uses the theory of binary relations and the utility theory.

In particular special features are in the fact that active systems are an object of problem- resource analysis; in the fact that the problem of subjective and objective factors separation is solved, in the fact that, as it will be seen from the following, the sufficiently definite and clear classification of resources is conducted, and resources are also considered as a subjective category.

Problems are subdivided into *qualitative* and *the quantitative*. Qualitative are such a problem, the result of solution of which has only two values: „yes" or „no" (respectively two numerical values: 0 and 1). For example, student either passed test or didn't (if he is interested just positive mark in). Meissner subjugated Everest or didn't, Amundsen reached the South Pole or no, and so on.

If S_σ is set of the permissible states of system, S_a is set of alternatives and A_σ is σ-algebra of subsets S_σ, then it is possible to determine preferences in the form

$$S_{a1} \langle S_{a2}; \quad S_{a1} \in A_\sigma; \quad S_{a2} \in A_\sigma.$$

1.2.2. Resources

Reasoning's in this and following paragraphs bear deterministic nature, although it is obvious that the chance essentially interferes behavior, conditions and results of an activity of active systems. The author base himself on the fact that accounting of randomness in this stage would complicate an account of basic sense and could be important only to obtain quantitative results.

Resources after the *problem* are the second most important category of problem- resource analysis.

Let's name resources any means and factors, which the subject consciously uses or will intend to use for the solution of its problems, or consider them as an expected result of the problems solution.

It would be tempting to present any *problem as the desired operation with resources: a conversion of some resources in others, a displacement (translation) of resources. A question consists in the following: is it possible to give to any problem this interpretation. A solution of any problem requires expenditures of resources. However, apparently there are problems, such that a result of solution of which is not new resources. For example, problems of enjoyment pleasures obtaining. Moreover, in certain cases, the solution of subject problem leads to decrease of its available resources. Indeed smoking, use of drugs reduces lifetime, i.e., decreases* „a quantity" *of available time of subject.*

In any case we can isolate a class of problems, *whose sense consists of* processing, converting *resources, or* displacement *of resources. At the same time there are problems, whose sense does not consists in obtaining* of new resources, *but of satisfaction of some important based from the point of view of the subject needs, moreover this result, whatever it is, cannot be subsequently used as the initial resources for the solution of the subsequent problems.*

A following reasoning is connected with the question distressing above. Let's assume that we deal with respect to the problem of second kind, when with the result of its solution is pleasure, enjoyment, generally, a satisfaction of personal needs of subject over a certain reasonable minimum, sufficient for maintenance and reproduction of his personal working and creative possibilities - personal potential. In this case it would be also possible to indicate, that satisfaction of such is an increased needs serves after stimulus for further activity, maintaining the will to further „navigation in the problem- resource ocean".

Because of that, we don't always identify the concept of state $\sigma_i \in S_a$ *of system with available resources.*

Examine definition of resources given above, that will be designated by symbol R. This definition connects resources with problems and with the subject. Since the problem (or alternative) is the subjective category, which exists in the consciousness of subject, than „resources" don't exist separately from the subject, and are also a subjective category. We want to say that some real objects, energy, information, labor, money become „resources", as soon as they in the consciousness of subject are connected with certain problem. For another subject they might not be resources, or to be the same, but in connection with completely different problem.

Thus, concept „resources" makes dual sense: on one side resources identify with ideas of subject about the fact that some objectively existing means and factors can be used for the solution of his problems and, from the other side - these are real means and factors (Fig. 1.2).

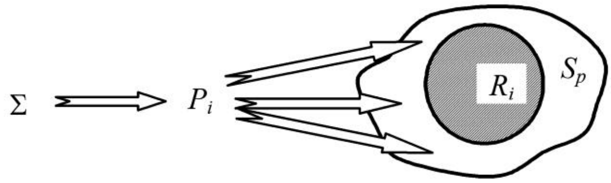


Figure 1.2

In a Fig 1.2 we want to show that the subject at first realizes problem P_i, and then basing on the set S_p of accessible means and factors selects some which necessary for solution of the problem P_i , then calls this resources R_i. If at the point of any reason he rejects solution of the problem P_i, then at the same moment selected means and factors cease to be service lives.

„Reserves" are the variety of resources, intended for the solution of problems of the certain type in the future. Let's correlate reserves with set of states alternatives S_a. Reserves let's designate S_p (Supply).

Consequently reserves, being the variety of resources, have a dual nature as well: as a subjective category and as the actually existing means and factors. We will suppose that studying reserves and set S_a (or W_a), the subject create a distribution of preferences. In this case „reserves" are transformed in resources, correlated to concrete problems.

For further analysis a classification of resources on a number of signs should be conducted.

Passive and active resources

With the solution of each problem, corresponding resources are used: finances, material, energy, information, which is available for the subject. In order to use these resources a subject is forced to act personally, i.e., to use his intellectual, physical service lives, his own time, which is for each subject the most important form of resources. The first of the mentioned forms of resources - „external" with respect to the subject - we will call passive resources R_p. The second type of resources (intellectual, physical, time of subject himself) - „internal" (endogenous) resources of subject name active resources R_a.

It is natured to presuppose that. The use of passive resources in the course of problem solution requires from the subject with the necessity to spend of certain part of his own active resources.

Thus, if R_{p1}^{req} - required passive resources, then required active resources R_a^{req} are determined by amount of passive resources, which subject uses for solution of his problem:

$$R_a^{req} = F\left(R_{p1}^{req} \ldots\right). \tag{1.4}$$

In some tasks the relationship between "speeds" of resources conversion is used.

Let $$v_a^{req} = \frac{dR_a^{req}}{dt}; \qquad v_p^{req} = \frac{dR_{p1}^{req}}{dt},$$

Then $$v_a^{req} = f' \, v_{p1}^{req},$$

where: ***f'*** is the operator of the resources conversion.

In the simplest case: $$f = \frac{\partial f\left(R_p^{req}\right)}{\partial R_p^{req}}.$$

In its turn, a capability of subject for active work in the course of his problems solution is ensured by the presence of sufficient active resources $R_a^{disp} \geq R_a^{req}$, where R_a^{disp} - available active resources. Their reproduction requires expenditures of certain quantity of passive resources. In other words, there is dependence

$$R_{p2}^{req} = g\left(R_a^{req}, \ldots\right). \tag{1.5}$$

Equation (1.4) would be written in the more correct form

$$R_a^{req} = f\left(R_{p1}^{req} + R_{p2}^{req}, \ldots\right). \tag{1.6}$$

The relations between „speeds" of a change in the resources can be examined analogously.

Relations (1.4) - (1.6) have only a symbolic sense and just explain existing connections of active and passive resources. Functions f and g are not determined and in the general case they're unknown; however, in certain cases, they take a concrete form.

The following indication of the resources classification is from the accounting of the multiplicity of their use. We will distinguish resources of single use (for example, tooth paste) and repeated use (for example, toothbrush). Subsequently of the resources of first type we will for the brevity call simply resources, and of the second type resources technologies.

Thus, **technology** - means resources of repeated usage that ensure the conversion of single use resources as the result of the problem solution. We will designate technologies through f.

By particular technology f_α^β , it is meant a technology of the type α resources conversion on the „entrance" of the problem solution process for the type β resource on the „output" from this process, i.e.:

$$f_\alpha^\beta\left(R_a\right) \to R_\beta.$$

In more general meaning we will talk about the technology of the resources conversion as the result of the problem solution: R^{exp} (in contrast to resources designating through R^{req}, R^{disp}).

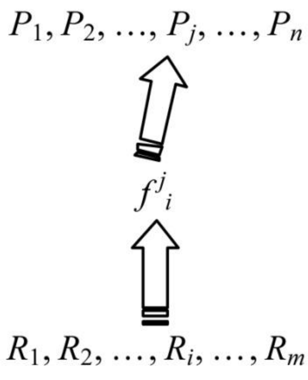


Figure 1.3

Let there be *m* different types of the resources: $R_1, R_2, \ldots, R_m$, and *n* different problems to solve: $P_1, P_2, \ldots, P_n$. Figure 1.3 illustrates the concept of the elementary technology f_i^j as the means of resources conversion of *i-th* kind to the result of the *j -th* of problem solution.

Fig. 1.4 shows schematically, what is implied by the packaged technology of the problem P_j solution, or simply by technology for the problem P_j:

$$F_j = \left(f_s^j, ..., f_i^j, ..., f_r^j\right). \tag{1.7}$$

It is obvious that F_j is not the sum of elementary technologies. Generally, it is doubtful that it is always possible *„to decompose"* the packaged technology on the elementary ones. Actually, the transformation of metallic billet (R_1) into the finished part requires the expenditures of electric power (R_2) and, therefore, in this case it is difficult to decompose the technology of components production on the elementary technologies. In certain cases this succeeds in making, if we present the process of the problem solution in the form of hierarchical procedure. This means that in this case it is necessary to disaggregate problem P to the hierarchically connected sub-problems. Let us designate as F_m^n the rectangular matrix $m\times n$ of the elementary technologies:

$$F_m^n = \begin{bmatrix} f_1^1 & f_1^2 & ... & f_1^n \\ f_2^1 & f_2^2 & ... & f_2^n \\ ... & ... & ... & ... \\ f_m^1 & f_m^2 & ... & f_m^n \end{bmatrix}. \tag{1.8}$$

Technologies, being the resources of repeated use, possess the property of physical and moral antiquating.

The separate class of problems can be described as the problems of creation of technologies, i.e., a creation of the resources of repeated use. In the economic applications production functions are examined.

We will use subsequently term technological functions *or* technological operators *for the designation of the procedures of the conversion of resources into the results of the permission of problems solutions.*

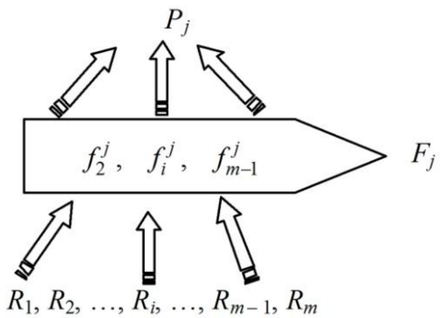


Figure 1.4

In the practical tasks the need for defining concretely the forms of resources appears. The roughest classification is the division of resources into material R_m, energy R_e and information R_{inf}. Generally speaking, each of these forms of resources are not encountered in the pure form. They are always mixed in a certain proportion. Thus, information resources have a material carrier, and their translation requires expenditures of energy.

In some applications, in Kobb - Douglas production function, for example, labor resources R_L (or L) and capital R_C (or C), are examined. Labor resources of hired workers, who are not a subject of this active system, are conveniently placed in the category of passive resources. We will carry labor resources of the system subject to the kind of active resources.

The most important kind of resources is money which in many cases appears universal. Precious metals and other values, that have features of money, are another form of universal resources.

Discussion the theories of money are not our task. Let’s make a note of only three important observations.

1. Not only money or treasures can posse’s universality. In the first years of passage to market relations in the countries of the CIS, the role of money was practically brought to zero, and barter transactions, when to one extent or another the properties of money were appropriated to different goods, were widely used.

The theory of money is presented, for example, in a classical book of Harris [154]. The role of money as information resources is examined in the book of Chernavskiy [159], including analysis of some dynamic models of changes in a monetary stock.

Any form of resources can be characterized with degree of their universality. If on the set S_a (or W_a) set of problems P_a , is assigned P_a^* is the subset of $P_a : P_a^* \subset P_a$ and some kind of resources can be used for solution of the problems $P \in P_a^*$, but it cannot be used for of the problems solution $P \notin P_a^*$ i.e. - on set the $P_a \backslash P_a^*$, then the degree of universality of such resources can be determined by any criterion, which characterizes relationships of sets P_a^* and P_a. In the simplest case it can be relation N^* / N , where N^* is number of different problems is P_a^*, and N - number of problems in P_a. It seems that the utility of resources of certain kind is determined not only by their consumer properties, but also by a degree of their universality.

2. Money in the sense of the classification given above can be attributed to information resources, since they contain information about previous activity of subject and about his potential possibilities in future in the concentrated impersonal form.
3. Not all problems can be solved, having available only money. Moreover, it is possible to give an infinite number of examples, when money, are not resources, they cannot be used, moreover not only in the past, but also due to conditions of today's peace. There are set of problems of this kind appearing before commanders during the war, different kind of creative problems, solved by scientists and other.

In connection with this it is possible to refer Paul S. Bragg's statement, made on another occasion, but the well illustrating position, formulated above: „At the point of money it is possible to purchase bed, but not sleep; food, but not appetite; medicine, but not health; building, but not domestic center; book, but not mind; adornment, but not beauty; luxury, but not culture...”

As we can see, in each of combination „left side” - these are real values, acquired at the point of money, i.e., something, objectively existing out of the subject and; „right side” - constitute subjective perceptions and sensation himself and the surrounding world.

Time and space as resources

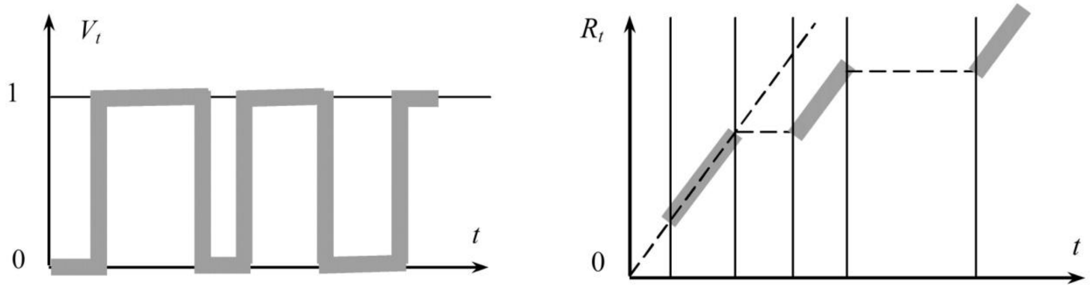
Time is the most important form of lives service. We will distinguish an *astronomical* time and an *operating* time. The astronomical times generally are not resources, and it will be used as an independent variable in the dynamic tasks. The operating time will be considered as service lives (quantity of training hours, an account of this discipline, quantity of hours of trainings of athlete, a time of flight operations of aircraft, so forth). The operating time R_t is a function of the astronomical time t:

$$R_t = R_t(t, \ldots).$$

Rate of an expense of operating time (temporal resources) (Fig. 1.5):

$$v_t = \frac{dR_t}{dt} = \begin{cases} 0; \\ 1. \end{cases}$$

Similarly we shall distinguish an astronomical space and an operating space effective area of shop, store, area for a sport game, volume of a camera of refrigerator and so forth. The operating space „is inscribed" in the astronomical space, its properties are determined not only by geometric dimensions, but also by the nature of the problem, for solution of which it is used.



a *b*

Figure 1.5

Available, required and expected resources

A classification of resources referred to their relation to the sense of the solved problem and sequence of actions of subject will be required for further constructions.

Following concepts will be used:

1. Available resources: $R^{disp} = R^{d}$. These resources at the given moment of time are in subject's disposal, and he has a capability to manage by them according to his discretion. If subject studies set P_a^* of problems, then he leans against „*his*" available resources. If these resources are universal for entire subset P_a^*, then we will consider that one and the same quantity of resources R^{disp} places as basis of an analysis of the solvability of all problems $P(\sigma_i)$. If resources R^{disp} are not universal, then a part of them are separated for each of problems $P(\sigma_i)$ from a total amount of available resources

$$R^{disp}(\sigma_i) \subseteq R^{disp},$$

which can be used for the solution of this specific problem?

2. To each specific problem $P(\sigma_i) \in P_a$ *required resources* $R^{req}(\sigma_i)$ are confronted. The determination *of required resources* is a task of forecast and is carried out by different methods (a statistical analysis of retrospective information, a determined calculation of expenditures, an expert estimation, ...).

We further assume that a confrontation of required and available resources makes it possible to form set of alternatives S_a (or W_a). In this case it is assumed that both forms of resources allow measuring them in comparable units. Then the problem $P(\sigma_i)$, for which

$$R^{req}(\sigma_i) < R^{disp}(\sigma_i), \tag{1.9}$$

must be included by the subject in the set S_a (or W_a), which in the theory of control is called the attainable set. Subsequently we will talk about an interpretation of properties of attainability and controllability in connection with problem- resource situations again.

3. Expected resources $R^{exp} = R^{e}$ are those, which the subject expects to obtain as a result of the problem solutions.

If a problem consists of a conversion of resources then, as it seems to us, a subject must exclude an examination of a problems, for which a condition $R^{exp}(\sigma_i) \le R^{req}(\sigma_i)$ is fulfilled and to examine such alternatives, for which

$$R^{exp}(\sigma_i) > R^{req}(\sigma_i). \tag{1.10}$$

Thus, the conditions for the including of a problem in the set P_a (alternatives - in the set S_a or W_a) are inequalities (1.9) and (1.10).

It is natural from the point of view of a comparison of the resources R^{disp}, R^{req} and R^{exp} to divide possible problems in two classes:

- the one of problems, which compose the set P_a^{α} in the case, if only first inequality (1.9) is fulfilled;
- the other of problems, which compose the set P_a^{β} if they carry out both inequalities (1.9) and (1.10). Set P_a^{α} is wider than set P_a^{β}:

$$P_a^{\beta} \subseteq P_a^{\alpha}.$$

In the set P_a^{α} such problems are contained, the result of solution of which is not new resources $R^{exp}(\sigma)$, but satisfaction of individual needs, when, let's say, it cannot be considered that active resources of subject adhere.

When the set of problems are not limited with inequalities (1.9) and (1.10) another view on the problem is possible, and it covers all accessible to an attention of the subject hypothetically possible states (alternative), which in principle can be realized, may be with a very small probability.

If we take a probabilistic point of view during an estimation of a reliability of alternatives, then set of alternative- possibilities studied by the subject would be substantially enlarged. Accordingly, the set of problems would be enlarged, if problem was formulated as follows: *„I am at the present time in state σ_a, but state σ_i, pleases me more, and in this case I don't think if, my resources will be sufficient, in order to reach state σ_i*, but *theoretically there is a possibility that under specific conditions in future, such resources can appear*".

The set of such problems $P_a^{(c)}$ is wider than the set of problems P_a, limited by inequalities (1.9) and (1.10):

$$P_a \subseteq P_a^{(c)} .$$

The set $P_a^{(c)}$ can reflect a wide range of a priori preferences of a subject, his tastes, from results of training, cultural level, ethnic, religious, political preferences.

A determination of a real resource basis of preferences separates pragmatic set of problems P_a and makes it possible „to include" quantitative methods for analysis of problem- resource situations.

1.2.3. Purposes

The next major category of problem-resource analysis is the category of „purpose". [105]. This category, in a first approximation is described in [64], where a appropriate definition of purpose, corresponding to what is understood in this case is given. We will not repeat the reasoning, given in already [64], just note the fundamental positions from our point of view.

The purpose is the intention, the decision to act in accordance with one of the existing alternatives in the problem-resource situation.

As you can see, the purpose - is a subjective category. The purpose should have a carrier - a subject. Target selection often is being outsourced to machine (rocket selects target, computer plays chess with world champion and wins, etc.) However, it is clear that the algorithm for selecting targets and to meet the criteria is transferred to the machine, the primary carrier of which is the subject.

In logical and temporary sense the *purpose* is a subordinated category. It can appear only since the *problem* is realized. Certainly, it could be imagine a certain chain of problems and purposes, when one problem, for example, P_0 generates the purpose A_0, in the process of motion to the purpose A_0 appear new intermediate problems P_1, P_2, ..., P_n and, correspondingly, new purposes A_1, A_2, ... However, in each individual case a *„problem"* is considered first, and the purpose - second.

The set of problems P_a would be possible to name *set of potential purposes*, and the selected problem for realization - *urgent purpose*. This, however, is a question of terminological agreement, but not a clue of the matter.

The selection of purpose is achieved as a result of analysis *of problem-resource situation*, i.e., the set of alternatives S_a (or W_a), available and required resources $R^{disp}(\sigma_i)$, $R^{req}(\sigma_i)$, and also expected new resources $R^{exp}(\sigma_i)$, and the distribution of preferences on S_a (W_a) or their Cartesian products.

The more detailed determination of the problem-resource situation within the framework of subjective analysis is proposed at p.1.3.

Purpose is represented as „an *operator*", the starting mechanism of reaching a selected state σ_i - conversion of resources.

The scheme proposed here puts problem in the first place, and assign to purpose the subordinate place.

1.3. Elements of theory of individual utility

1.3.1. Binary relations, ordering

It was already said, that the utility theory can be used as the basis, while studying the active systems. The simplest problem is defined as preference relation on the set of alternatives. The utility theory gives the model of the preferences distribution forming. The utility problem- resource analysis is not the precise copy of the theory. The latter, is built within the framework of ordinal approach; while the subjective analysis of problem- resource situations does not exclude cardinal approach, calculation of ethical factors and others.

It is known that early marginal's (followers of the margined utility theory) considered that subjective tastes and preferences on set specific collections of goods is determined *by the cardinal* utility, which can take any numerical values from the given set. This function is considered as the cardinal measure of the satisfactoriness of user. It is implied that the user can assign to this measure arbitrary numerical values and, thus, determine his preferences [199].

At the point of the change *to cardinalism, the* alien another point of view, was taken. It is from the persuasion, that the consumer at best, evaluating the utility of the different collections of goods, can determine their order, i.e., speaking in modern language, to establish binary relation ρ: $\rangle$, $\langle$, $\sim$.

The corresponding direction was called *ordinalism*, and the preferences distribution - ordinal distribution. One of the arguments in favor of *ordinalism* is the absence of the reliable means of measurement and, which especially important, prediction of subjective preferences. It is supposed that each user can say: „This collection of goods, pleases to me more (less, equivalent) than another collection of goods". However, passage to the *ordinalism* gave birth to the large number of logical and mathematical difficulties, especially, when it goes on about collective preferences. The getting over these difficulties is frequently represented in the form „the theorems about impossibility". There are reasons for the criticism of *ordinals*, as well as, *cardinalism*, but from the different positions.

The theory, which as particular cases contains both cardinal and ordinal versions, is preferable.

The postulation *of the variational principle*, in correspondence with which the preferences are formed and, therefore, - the property of the optimality of the corresponding mental processes (chapter 3) is essential in the present work.

Variational principle, and its modifications, leads to the so-called *canonical distributions of the preferences*, which depend on such quantitative characteristics as resources of different types, the quantitatively determined utility and harmfulness.

The canonical distributions give the analyst the promising and flexible apparatus of quantitative analysis. A deficiency in the experimental data is completed by the postulation of qualitative principle. The concepts *of subjective entropy* and *subjective information* that play exceptional role are introduced.

Thus, return to the *cardinal* position on qualitatively new level gives a number of advantages. Preferences are subdivided as the rationalistic, connected with the utilitarian interests (utility, harmfulness, resources,...) and irrationals (stable imperatives, ethical, religious, political,...). In this case the assumption is done that if the rationalistic preferences, are object of optimization („in the depths of psyche") on the basis of variational principle mentioned above, then stable imperatives are assigned by a -priori and cannot be the object of the selection, including of optimum. They are the result of past accumulated experience.

One of the arguments in favor of the use of cardinalism in the contemporal conditions is the presence of powerful „assistant" for decision making, such as computer technologies are - different systems of decision making support, in principle capable with no matter how small „step" of discrediting alternatives with respect to the characteristics of their effectiveness.

Truly immense literature is dedicated to the theory of utility. The axiomatic method of this theory constructing adapts by Ramsay, Neumann and Morgenshteyn, by Sevitsh, Debra at al. We give the very selective and to a considerable extent random list of some works in this region. For future reference it will be useful to give the brief enumeration of basic determinations and assertions. We will use for this purpose the summary-type reporting following the book Fishbourn [149] and work [27].

The theory of utility is from the mathematical theory of binary relations, as a basis of which the concept of the preferences lies and which in turn rests on the more general theory of categories [96].

There are, at least, two variations of the utility theory. In the first version probabilistic ideas about the utility are not used, in the second the uncertainty, expressed through the probability, is considered. The corresponding theory bears the name of the expected utility theory.

Subsequently we will not repeat references each time. Retreats from the mentioned sources will be specified. In the account we omit the proofs of theorems, and partially change designations, taking into account the use of various symbols in this book for other purposes. In particular binary relation is designated by letter ρ, since through R everywhere subsequently we will designate resources. Set of alternatives we will designate through S_a, alternatives - through σ, φ, ξ,... or σ_i, σ_j, σ_k,...

Arbitrary binary relation is designated through ρ, and the relation of ordering by symbol „$\langle$". Symbol „$\Rightarrow$" indicates „it draws". Symbol „$\Leftrightarrow$" indicates „then and only then".

Relation $\sigma \langle \eta$ is called weak relative ordering, if

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta), \tag{1.11}$$

where U (.) - the function of utility.
Relation $\sigma \langle \eta$ is strict partial ordering if

$$\sigma \langle \eta \Rightarrow U(\sigma) < U(\eta). \tag{1.12}$$

Let's enumerate properties, which the binary relations can possess (symbol «$\forall$» is read „for all", symbol «$\exists$» indicates „there exists").

1. Relation ρ is reflexive, if $\sigma\rho\sigma$ for $\forall\sigma \in S_a$.
2. Relation ρ is non-reflexive, if $\sigma\bar{\rho}\sigma$ for $\forall\sigma \in S_a$.
3. Relation ρ is symmetrical, if $\sigma\rho\eta \Rightarrow \eta\rho\sigma$ for $\forall\sigma, \eta \in S$.

For example, if „σ is a brother η", then „η is a brother σ ".

4. Relation ρ is asymmetric, if $\sigma\rho\eta \Rightarrow \eta\bar{\rho}\sigma$ for $\forall\sigma, \eta \in S_a$.

Here „ $\bar{\rho}$ " indicates „not to be available in this sense".
In this case, for example, if „I prefer η in comparison with σ, then σ is not preferable with respect to η":

$$\sigma \langle \eta \Rightarrow \eta \bar{\langle} \sigma. \tag{1.13}$$

5. Relation ρ is antisymmetric, if $(\sigma\rho\eta, \eta\rho\sigma) \Rightarrow \sigma = \eta$ for $\forall\sigma, \eta \in S$.
6. Relation ρ is transitive, if $(\sigma\rho\eta, \eta\rho\xi) \Rightarrow \sigma\rho\xi$ for $\forall\sigma, \eta, \xi \in S_a$.
7. Relation ρ is negatively transitive, if $\left(\eta\bar{\rho}\sigma, \eta\rho\xi\right) \Rightarrow \sigma\rho\xi$ for $\forall\sigma, \eta, \xi \in S_a$.
8. Relation ρ is called connected or complete, if for $\forall(\sigma, \eta) \in S_a$ occurs either $\eta\rho\sigma$ or $\sigma\rho\eta$.
9. Relation ρ is called weakly connected, if for $\forall\sigma, \eta \in S_a$ and $\sigma \neq \eta$ occurs either $\eta\rho\sigma$ or $\sigma\rho\eta$.

Binary relation ρ introduces in S_a *the weak ordering*, when it is asymmetric and it is negatively transitive.
Relation ρ introduces is S_a *a strict ordering*, $\Leftrightarrow$ if it is weakly connected (see p.9) and weakly ordered.
Relation ρ is relation of equivalence (~) $\Leftrightarrow$, if it *is reflexive* (p.1), *transitive* (p.6) and *symmetrical* (p.3).
Relation of equivalence $\rho = \sim$ assigns the partition of set S on the classes of equivalence $S(\sigma)$. Symbol „~" is defined as the relation of indifference. Class $S(\sigma)$ catches by element σ, $\forall\sigma \in S_a$, has its class of equivalence, an in particular can be consisting of one element - very σ, the set of all classes of equivalence we'll designate through $S_\sim$.
The following table in compact form characterizes forms of ordering described above.

<u>ρ introduces in S_a weak ordering $\Leftrightarrow$ when it</u>
1) is asymmetric: $\sigma\rho\eta \Rightarrow \eta\bar{\rho}\sigma$ for $\forall(\sigma_1\eta) \in S_a$;
2) is negatively transitive: $\left(\eta\bar{\rho}\sigma, \eta\rho\xi\right) \Rightarrow \sigma\rho\xi$.

<u>ρ introduces in S_a a strict ordering $\Leftrightarrow$ when</u>
1) between the elements S_a is a weak connection: for $\forall\eta, \sigma \in S_a, \eta \neq \sigma \Rightarrow \sigma\rho\eta$ or $\eta\rho\sigma$.
2) occurs weak ordering $\Rightarrow$

$$\Rightarrow \begin{cases} \text{asymmetry;} \\ \text{negative transivity.} \end{cases}$$

<u>ρ is equivalence relation (~) $\Leftrightarrow$ when it</u>
1) is reflexive: $\eta\rho\eta$, $\forall\eta \in S_a$;
2) is symmetrical: $\eta\rho\sigma$, $\forall\sigma, \eta \in S_a$;
3) is transitive: $(\eta\rho\sigma, \xi\rho\eta) \Rightarrow \xi\rho\sigma$; $\forall\sigma, \eta, \xi \in S_a$.

<u>ρ introduces in S_a a strict partial ordering $\Leftrightarrow$ it</u>
1) is non-reflexive: $\sigma\bar{\rho}\sigma$, $\forall\sigma \in S_a$;
2) is transitive.

The example of negative tranistivity are the relation, installed by assertion „who to us it not enemy, then he is our friend". The relation of seniority is an example of asymmetric relation. If $\eta\rho\sigma$ means that „σ is a son", and „η is the father", then must be $\eta\bar{\rho}\sigma$ („η is not son for σ ").

From the set of all possible binary relations preference relation will interest us, for which the symbol $\langle$ is used:

$$\rho = \langle .$$

Below we will study the properties of the binary preferences relation. Let's give some theorems and definitions.

Theorem 1

Let $\langle$ - weak ordering on S_a, then:
1) for $\forall\sigma, \eta \in S_a$ occurs at least one of the relationships:

$$\eta \langle \sigma;\ \sigma \langle \eta;\ \sigma \sim \eta;$$

2) relation ~ is equivalence, i.e., reflexive, symmetrical and transitive;
3) relation $\langle$ is transitive and connected;
4) from ($\sigma \langle \eta$, $\eta \sim \xi$) $\Rightarrow \sigma \langle \xi$;
5) let $S_\sim$ is set of the classes of equivalence on S_a, and $\langle_\sim$ - preference relation on the set of classes $S_\sim$. Relation $\langle_\sim$ is determined by the condition:

$$S_\sim(\sigma) \langle_\sim S_\sim(\eta) \Leftrightarrow$$

when can be found elements $\sigma \in S_\sim(\sigma)$ and $\eta \in S_\sim(\eta)$ such, that $\sigma \langle \eta$. Then relation $\langle_\sim$ on $S_\sim$ is a strict ordering.

Theorem 2

If relation $\langle$ on S_a is weak ordering, $S_\sim$ is countable set, *then there is a real function U*, S_a, for which:

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta);\ \forall\sigma, \eta \in S_a.$$

Relation $\sigma \approx \eta$ is introduced, which is carried out when and only when condition ($\sigma \sim \xi\ \eta \Leftrightarrow \sim \xi$) are satisfied for $\forall\xi \in S_a$. Here equivalence is determined by the comparison of elements σ and η with the third element ξ, which must exist at S_a.

Theorem 3

If $\langle$ on S_a is strict partial ordering, then
1) for $\forall\sigma, \eta \in S_a$ one of the relations $\sigma \langle \eta$; $\eta \langle \sigma$; $\sigma \approx \eta$ ($\sigma \sim \eta \wedge \overline{\eta \approx \sigma}$) is fulfilled;
2) relation $\approx$ is relation of equivalence;
3) $\sigma \approx \eta \Leftrightarrow$ ($\sigma \langle \xi \Leftrightarrow \eta \langle \xi$ and $\xi \langle \sigma \Leftrightarrow \xi \langle \eta$) for $\forall\xi \in S_a$;
4) ($\sigma \langle \eta$; $\eta \approx \xi$) $\Rightarrow \sigma \langle \xi$ and ($\sigma \approx \eta$; $\eta \langle \xi$) $\Rightarrow \sigma \langle \xi$;
5) let $\langle^*$ - preference relation on the set of the classes of equivalence $S_\approx$ (where $\approx$ is relation of equivalence, defined above) such, that for $\forall S_1, S_2 \subset S_\approx$, $S_1 \langle^* S_2 \Leftrightarrow$will be found such elements $\sigma \in S_1$ and $\eta \in S_2$, that $\sigma \langle \eta$. In this case $\langle^*$ is strict partial ordering on $S_\approx$.

In the utility theory essential role plays Zorn's lemma:

Let ρ is strict partial ordering on S_a and for $\forall Z \subset S_a$, on which ρ - strict ordering, there is an element $\sigma \in S_a$ such, that for $\forall\xi \in Z$ the condition $\xi\rho\sigma$ or $\sigma = \xi$ is satisfied. Then such element $\eta^* \in S_a$, for which $\eta^*\rho\xi$ doesn't carry out for any $\xi \in S_a$ can be found.

1.3.2. Determinate utility

The following theorem introduces the function of utility $U(\sigma)$ if relation $\langle$ on S_a is strict partial ordering, and the set of the classes of equivalence $S_\approx$ is countable . Then, according to theorem, there is a real function U on S_a such, that for $\forall\sigma, \eta \in S_a$

$$\sigma \langle \eta \Rightarrow U(\sigma) < U(\eta); \tag{1.14}$$

$$\sigma \approx \eta \Rightarrow U(\sigma) = U(\eta). \tag{1.15}$$

In connection with the consideration of preference relation the question about the distingishability of alternatives naturally arises.

Theorem 2 bases existence of the utility function, when relation $\langle$ makes an weak ordering on S_a. The following theorem makes it possible to conduct the utility function in the case, when relation $\langle$ corresponds to a strict partial ordering on S_a.

Theorem 4

If relation $\langle$ is strict partial ordering on S_a, set of subsets of equivalence $S_{\approx}$ are countable, then there is a real function $U(\sigma)$, assigned on S_a such, that for $\forall\sigma, \eta \in S_a$ (1.14) and (1.15) are carried out. The important expansion of the theory given above is the case of interval ordering. The regulated indifference intervals do not coincide with the classes of equivalence and reflect the properties of the psyche of subject that for him there are zones of the indistinguishability of alternatives.

Let's examine the additional conditions, which introduce alternative ordering, but in this case they preserve the property of the intransitivity of the relation of indifference (~):

$$(\eta \sim \sigma; \xi \sim \eta), \text{but } \overline{\sigma \sim \xi}.$$

Let's add to the conditions of theorem 3 following conditions:

$$(\sigma \langle \eta; \xi \langle \varepsilon) \Rightarrow (\sigma \langle \varepsilon \text{ or } \xi \langle \eta) \text{ for } \forall\sigma, \eta, \xi, \varepsilon \in S_\sigma; \tag{1.16}$$

$$(\sigma \langle \eta; \eta \langle \xi) \Rightarrow (\sigma \langle \varepsilon \text{ or } \varepsilon \langle \xi) \text{ for } \forall\sigma, \eta, \xi, \varepsilon \in S_\sigma; \tag{1.17}$$

The sense of these conditions can be illustrated graphically if we assume that elements σ, η, ξ, ε - real numbers (Fig 1.6).

It is evident that, if relation $\langle$ is non-reflexive and, furthermore, two conditions, recorded above are satisfied, then relation $\langle$ is transitive.

Relation $\langle$ is called *interval ordering*; if it is nonreflexive and possesses property (1.16). But if additionally property (1.17), is carried out, then it is called *interval semi-ordering*.

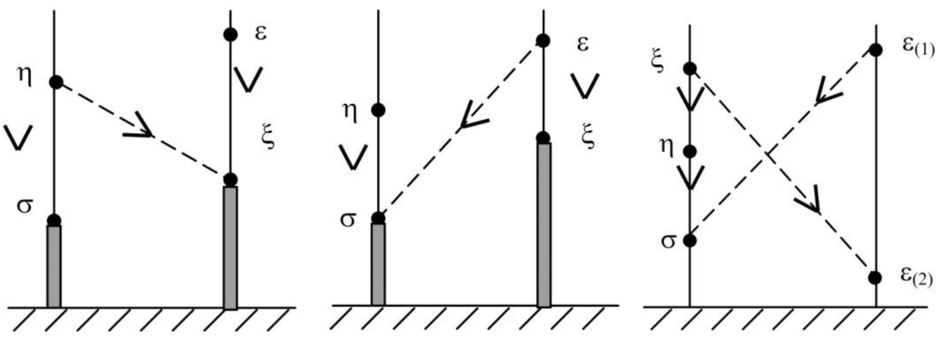


Figure 1.6

If relation $\langle$ on S_a is interval ordering, and $S_{\approx}$ is countable, then there are real functions $U(\sigma)$ and $\delta(\sigma)$, assigned on S_a such, that $\delta(\sigma) > 0$ for $\forall\sigma \in S_a$, and

$$\sigma \langle \eta \Leftrightarrow U(\sigma) + \delta(\sigma) < U(\eta) \text{ for } \forall\sigma_1\eta \in S_a. \tag{1.18}$$

This theorem introduces *function of uncertainty* $\delta(\sigma)$, therefore the retention of the intransitivity of the indifference relation «~» is possible. Indifference interval:

$$I(\sigma) = [U(\sigma), U(\sigma) + \delta(s)]. \tag{1.19}$$

The indifference interval $I(\sigma)$ lies entirely to the left of the interval $I(\eta)$ if, and only if $\sigma \langle \eta$.

If the intervals $I(\sigma)$ and $I(\eta)$ intersect, correspondent elements $\sigma, \eta \in S_a$ are located with respect to indifference.

The following theorem establishes indifference intervals for the case of semi-ordering on S_a.

Theorem 6

Let $\langle$ is semi-ordering on S_a and set of subsets of equivalence $S_{\approx}$ is finite.
Then on S_a *there* is a real function $U(\sigma)$ such, that

$$\sigma \langle \eta \Leftrightarrow U(\sigma)+1<U(h) \text{ for } \forall \sigma, \eta \in S_a. \tag{1.20}$$

Instead of one there can be any finite number. As it is evident in this case indifference intervals can have identical length.

The theory of utility on the non-countable sets is based on the concept of the density of alternatives set relatively ordered.

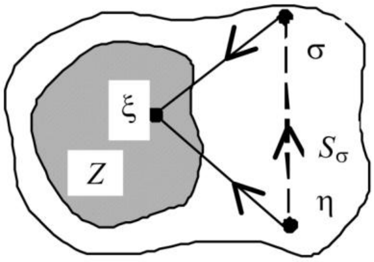


Figure 1.7

Definition

Let ρ is the binary relation on S_a, moreover S_a - non-countable set. Set $Z \subseteq S_a$ is called ρ - dense in S_a, if $\forall \sigma, \eta \in S_a$, not belonging at the same time to Z: ($\sigma \overline{\in} Z; \eta \overline{\in} Z$) and for which σρη such $\xi \in Z$, that (σρ ξ, ξρη) can be found (Fig.1.7). Assume that the relation $\langle_{\sim}$ is determined on the set of the classes of equivalence $S_{\sim}$.

Theorem 7

There exist such real function U(σ) on S_a, that the equivalence $\sigma \langle \eta \Leftrightarrow U(\sigma) < < U(\eta)$ for $\forall \sigma, \eta \in S_a$ occurs then and only then, when preference $\langle$ on S_a introduces weak ordering and set $S_{\sim}$ separable relative to the ordering $\langle_{\sim}$.

Set is called separable, if there is everywhere dense countable set in it [149].

Set *A* is called everywhere dense, in the set *B* if $\overline{A}$ coincides with the set *B*.

If preference relation $\langle$ is introduces on S_a a strict partial ordering, then the following theorem occurs.

Theorem 8

Let $\langle$ on S_a is strict partial ordering, and $\langle_{\approx}$ separable subset of the set of the equivalence classes $S_{\approx}$ exists.

Then there is an assigned on S_a real function $U(\sigma)$ such, that

$$\left.\begin{array}{l}\sigma \langle \eta \Rightarrow U(\sigma) < U(\eta) \\ \sigma \approx \eta \Rightarrow U(\sigma) = U(\eta)\end{array}\right\} \forall \sigma, \eta \in S_a.$$

The condition of separability is here sufficient, but not necessary.

Existence of the utility function $U(\sigma)$ in the case of the increasing preferences and weak ordering is established by the following theorem.

Theorem 9

Let S_a is „rectangle", the Cartesian product of the sets:

$$S_a = S_{a1} \times S_{a2} \times \ldots \quad \times S_{an}$$

in the space R^n, quantitative sense to symbols σ, η,... is assigned and for $\forall \sigma \in S_a$ the conditions are satisfied:

1. Relation $\langle$ on S_a is weak ordering.
2. $\sigma < \eta \Rightarrow \sigma \langle \eta$.
3. $(\sigma \langle \eta;\ \eta \langle \xi) \Rightarrow$ there are such $\alpha, \beta \in (0, 1)$, that $\alpha\sigma + (1 - \alpha)\xi \langle \eta;\ \eta \langle \beta\sigma + (1 - \beta)\xi$.

Then there is a real function $U(\sigma)$ on S_a, which satisfies the condition:

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta) \text{ for } \forall \sigma, \eta \in S_a.$$

Condition 2 is called the condition of monotonicity or non-saturation and means that the preferences grow with any increase in the quantity. Condition 3 is called Archimedes condition and is used while establishing of separable subset relative to ordering $\langle$ is made.

It follows from the previous theorem that, if $\sigma, \eta, \xi \in S_a$ and $\sigma < \eta < \xi$, then there is exactly one such $\alpha \in (1, 0)$, that

$$\eta \sim \alpha\sigma + (1 - \alpha)\xi .$$

In the case of the non-decreasing preferences and strict partial ordering the existence of function $U(\sigma)$ is established by the following theorem.

Theorem 10

Let S_a is non-negative octant in R^n and on S_a the following conditions are satisfied:

1. Relation $\langle$ on S_a - strict partial ordering.
2. $[(\sigma < \eta;\ \eta \langle \xi) \text{ or } (\sigma \langle \eta;\ \eta < \xi)] \Rightarrow \sigma \langle \xi$.
3. $\sigma \langle \eta \Rightarrow \xi < \eta$ for $\forall \xi$, that $\sigma < \xi$.

Then there is a real function $U(\sigma)$ on S_a, which satisfies the condition:

$$\sigma \langle \eta \Rightarrow U(\sigma) < U(\sigma) \text{ for } \forall \sigma, \eta \in S_a.$$

With these conditions $\sigma < \eta \Rightarrow \eta \langle \sigma$, which indicates, that an increase in the quantity does not decrease preference.

In [149] lexicographical orderings with respect to the preferences are examined when density condition is disrupted, and in the case of the strictly partial ordering the separability is sufficient, but not necessary for existence of real function of utility.

Additive utilities on the finite sets are determined by the relation:

$$\sigma \langle \eta \Leftrightarrow U_1(\sigma_1) + \ldots + U_n(\sigma_n) < U_1(\eta_1) + \ldots + U_n(\eta_n), \tag{1.21}$$

where each alternative σ = (σ_1, σ_2, ..., σ_n) is the „vector" - the element of the subset of the Cartesian product:

$$S_a = \prod S_{ai} = S_{a1} \times S_{a2} \times \ldots \times S_{an} .$$

The previous relation assumes the absence of correlation of the factors - components of vector σ. Preferences on the Cartesian degrees of the sets are examined:

$$S_a^{(n)} = \underbrace{S_a \times S_a \times S_a \times \ldots \times S_a}_{n},$$

where n – is the number of time periods.

The introduction of time factor makes it possible to examine the dynamic processes of a change of the utilities with time. The concepts formulated in the temporal aspect are studied: „persistence", „impatience", „discounting". Investigation in this direction belongs to Kushlen, Diamond, Williamson and others [149].

1.3.3. Expected utility

The important division of the utility theory relates to the so-called *expected utilities*, connected with the orderings on the sets of probability measures. The expected utilities were studied by Neumann and Morgenstern and, somewhat previously by Ramsay. Further modifications of theory belong to Friedman, Sewich, Kersten, Milnor , Cramer, Reiff, Blackwell, Girschik, Lewis.
Bibliographical references to the work of the authors mentioned above can be found in Fishburn [149].
Let's examine briefly some concepts of the theory *of the expected utilities*.
Simple probability measure P on the set X is assigned by conditions:
1. $P(A) \geq 0$ for $\forall A \subseteq X$. (1.22)
2. $P(X) = 1$.
3. $P(A \cup B) = P(A) + P(B)$, if A, $B \subseteq X$ and $A \cap B = \varnothing$.
4. $P(\varnothing) = 0$.
The mathematical expectation of function f for the countable or final set X is determined by the formula

$$E(f,P) = \sum_{x \in X} f(x) P(x), \tag{1.23}$$

where: $x \in X$ is elements of the set X. Sum is apply on all elements of set and the following theorem occurs.

Theorem 11

If P is simple probability measure, then $P(x) = 0$ for $\forall x_i \in X$, besides of the finite number of values x_i and

$$P(A) = \sum_{x_i \in A} P(x_i) \text{ for } \forall A \subseteq X . \tag{1.24}$$

If measure is convex, then the condition:

$$E\left[f, \alpha P + (1-\alpha) Q\right] = \alpha E(f, P) + (1-a) E(f, Q), \tag{1.25}$$

is satisfied;

where $\alpha \in (0, 1)$, P, Q are simple probability measures.

Theorem 12

Let P_a is the set of all simple probability measures on S_a and $\langle$ is the binary relation of preference on set P_σ. Them for existing a real function $U(\sigma)$ on S_a and feasibility of condition:

$$P \langle Q \Leftrightarrow E\left(U, P\right) < E\left(U, Q\right) \text{ for } \forall P, Q \in P_a \tag{1.26}$$

it is necessary and sufficient, that for $\forall P, Q, R \in P_a$:

1. relation $\langle$ was weak ordering on P_a.
2. $(P \langle Q, 0 < \alpha < 1) \Rightarrow \alpha P + (1-\alpha)R \langle \alpha Q + (1-\alpha)R$.
3. $(P \langle Q, Q \langle R) \Rightarrow (\alpha P + (1-\alpha)R \langle Q$ and $Q \langle \beta P + (1-\beta)R)$ for some $\alpha, \beta \in [0, 1]$.

Positive function $U(\sigma)$, assigned on S_a, is called the utility function and is unique with an accuracy to the positive linear transformation. This means that the function $V(\sigma) = aU(\sigma) + b$, where $a > 0$, satisfies the condition:

$$P \langle Q \Leftrightarrow E(v, P) < E(v, Q) \text{ for } \forall P, Q \in P_a, \forall \sigma \in S_a. \tag{1.27}$$

In the case of the additive expected utility, if P is a collection of measures P_i and Q is collection of measures $Q_i\left(i \in \overline{1, n}\right)$, the condition is satisfied

$$P \langle Q \Leftrightarrow \sum_{i=1}^{n} E\left(U_i, P_i\right) < \sum_{i=1}^{n} E\left(U_i, Q_i\right), \tag{1.28}$$

where: $U = (U_1, U_2, \ldots, U_n)$.

The interdependent expectations are studied. Important development is the theory of expected utility.

The survey of some facts of the utility theory, including a number of the theorems, which base existence of the utility functions, was given above following [149].

1.4. Further analysis of the fundamental notions.

1.4.1. Problem- resource situation

The brief account of the theory of binary relations and elements of the utility theory and also some concepts of problem- resource approach given above makes it possible to carry out further concrete definition of concept „active system", defining its distinctive properties.

1. Active system can exists being isolated, and to evolve thus far the available resources will not be exhausted; however, it tends to the interaction with „the environment", organizes this interaction, creating the flows of substance, energy, information by forming different gradients.
2. Assigned external effect on system causes the ambiguous reaction (response) of system. This most likely is manifested in the fact that

a) the distribution of preferences on the set of alternatives S_a, formed with system on this set changes;

b) set of alternatives S_a, changes, new alternatives appear, either the alternatives existed previously disappear or on a and another occurs at the same time.

3. Interacting with „the environment", system itself selects strategy. This strategy is not correlated unambiguously with the state of environment, and contains „remnant's" or spontaneous component.

4. System has „its" individual („build it") criterion of the optimality (better to say „rationality" or „effectiveness") - „sewn" in the consciousness of subject - in „the central control of a system". Most frequently this criterion bears the nature of Paretto criterion, when the solution is selected based on the unimprovable set of the values of vector criterion.
5. Any external action system converts as a certain set of „*its*" own problems. But most important is the fact that the active system „spontaneously" generates „*its*" problems. Some problems are solute, part is not.

Active system manifests effusiveness on a set of the admissible states SA, and forms set of alternatives S_a by itself. Thus, the required and formal attributes of active system are:

- the set of principally possible (admissible) states *SA* is the objective characteristic of system (for example, man cannot exists at a temperature of the body 45° C); for each realized state σ_0 at the given instant t - set of the attainable states $S_{att}|\sigma_0$ and set of alternatives $S_a|\sigma_0$. This latter is subjective characteristic of active system. $S_a|\sigma_0$. can coincide with $S_{att}|\sigma_0$;
- the presence on the set $S_a|\sigma_0$ preference relation ρ;
- the presence of set of problems $P_a|\sigma_0$ in each initial state.

All problems are divided as those solvable and insoluble. We will assume for the purpose of simplification in the theory that the insoluble problems are rejected immediately at first stages of analysis and the selection of the most preferable problem is achieved among the solvable problems.

The characterization of initial „state" σ_0 includes the available (available) resources R_0^{disp}, the required resources $R^{req}\left(\sigma_i\right)=R_i^{req}$ and the expected effect, i.e., the new resources as a result of the solution of the problem $R^{exp}\left(\sigma_i\right)=R_i^{exp}$. Alternative assumes the forthcoming action: passage $\sigma_0 \to \sigma_i$.

„Event" s is characterized as a „state" plus moment of time t: s: = (σ_0, t). „Situation" is characterized as an „event" plus set of alternatives $S_a|\sigma_0$. In the characterization of situation the resources R^{disp}, R^{req}, R^{exp} are included. Into the number of resources the available time t^{disp} and the required time t^{req} have to be included. In our case we will speak about „problem-resource situation"

$$m:=\left[\sigma_0, t, S_a \,|\, \sigma_0, \rho\,, R^{\mathrm{disp}}, R^{\mathrm{req}}\,(S_a \,|\, \sigma_0)\right],$$

where R^{req} is set of the values of required resources on $S_a|\sigma_0$, ρ preference relation.

The subject of on active system is situated continuously in a problem- resource situation (PRS). Alternation of PRS is defined as a „situation dynamics" (SD).

Set of problems $P_a|\sigma_0$ is introduced as correspondence $(\sigma_0 \to \sigma_i)\,\tilde{\rho}\,(R_i|\sigma_0)$ between the alternatives and the resources. Here by resources vectors $R_i|\sigma_0$: = (R^{disp}, $R^{req}(\sigma_i)$, $R^{exp}(\sigma_i)$) are implied. If alternative makes purely economic sense, then the determination „directed" preference is connected with the comparison of the expenditures $R^{req}(\sigma_i)$ and the expected effect $R^{exp}(\sigma_i)$.

Problem $P_i|\sigma_0$ is considered solvable, if the conditions $R^{req}(\sigma_i) < R_0^{disp}$ is satisfied and insoluble, if $R^{req}(\sigma_i) \geq R^{disp}$ are satisfied. Among the solvable problems the part of the problems are effective. These are such problems, for which the inequality $R^{req}(\sigma_i) < R^{exp}(\sigma_i)$ is fulfilled.

For the ineffective problems $R^{req}(\sigma_i) \geq R^{exp}(\sigma_i)$.

1.4.2. Situational dynamics

By the „situational dynamics" from one side we understood the process of changing the problem-resource situation, from another side - the mathematical description of this process.

All elements of system undergo chronological variation. The realized state of system changes together with time, the set of alternatives $S_a|\sigma_0$, preference relation $\rho(S_a|\sigma_0)$, the available resources $R^{disp}(\sigma_0)$, and also the required resources, assigned for the change of alternatives $\sigma_i \in S_a|\sigma_0$. Let's note that as

„the limitations" in the course of alternatives selection can come out not only the integral available resources, but also the maximum speeds of their use (expense, conversion).

To describe dynamics PRS m — it means to determine the system of the mathematical relationships, which describe chronological variation of all elements, which are contained in the formal description of situation.

„Being moved" in the time, system changes its actual state („point of view") σ_0. Simultaneously in connection with a change of „point of view" and a change in the available resources $R^{disp}(\sigma_0)$, modification of the set of alternatives $S_a|\sigma_0$ occurs. In view of the action of internal and external factors (for example, change in the economic situation, laws, prices and others) a change in the required resources $R^{req}(\sigma_i)$, $\sigma_i \in S_a|\sigma_0$ occurs.

Finally taking into account entire aforesaid, the distribution of preferences on the set $S_a|\sigma_0$ changes. Subsequently to each preference relation quantitative characteristic - distribution of preferences $\pi(\sigma_i|\sigma_0)$ is placed in the correspondence. Cardinal approach to the forming of preferences adapts. We will examine the cases, when there is a correspondence,

$$\rho(S_a \mid \sigma_0) \Leftrightarrow \pi(S_a \mid \sigma_0), \tag{1.29}$$

and from (1.29) it follows that

$$\sigma_i \underset{\sim}{<} \sigma_j \,|_{\sigma_0} \Leftrightarrow \pi\left(\sigma_i \mid \sigma_0\right) \le \pi\left(\sigma_j \mid \sigma_0\right). \tag{1.30}$$

In the case, when $\sigma_0 \,\overline{\in}\, S_a \,|_{\sigma_0}$, the function of preferences we will designate $\pi(\sigma_i)$ and call „absolute" or „non- conditional". To assign the description of situation dynamics for the active system - means to assign conversion (morphism):

$$m(t_2) = F\left[m\left(t_1\right)\right], \tag{1.31}$$

or

$$M\left(t_2\right) = F\left[M\left(t_1\right)\right], \tag{1.32}$$

where $M(t_1) \in M_a(t_1)$ and $M(t_2) \in M_a(t_2)$, $M_a(t)$ is set of alternative PRS, and M(t) is subset PRS. Depending on the accepted model of description morphism (1.32) can be assigned in a various forms: as the relation, probabilistic or illegible relation. In the course of time the function of preferences $\pi(\sigma_i|\sigma_0)$ is modified. Since by hypothesis that lifetime of each active system is finite, than $(t_1, t_2) \in [t_0, t_0 + t_f]$, where t_f —time „of life" of system.

In the simplest case transformation (1.31) is represented as the system of ordinary differential equations, or as the system of ordinary differential equations and equations in partial derivatives.

If in a change of the situation population processes, transmitting of imperious authorities, or processes of instruction and some other specific processes, which possess „memory", participate, then mathematical model includes integro-differential equations.

The description of situation m includes $S_a|\sigma_0$, $R^{req}(S_a|\sigma_0)$, $R^{disp}(S_a|\sigma_0)$,..., therefore the model of dynamics must reflect the dynamics of a change in these sets. This requires the attraction of the corresponding methods.

1.4.3. Dynamics of sets and derivative with respect to measure

One of the possible approaches is the use of a concept of set's function [16].

Let E is the set, $E' \subset E$, and U(E) is set of subsets E'. Assume that to $\forall E' \in U(E)$ the number $x_{E'} \in R$ is placed in the correspondence, where R is set of real numbers.
Let's designate this correspondence *F*.

$$F : U(E) \to R \,. \tag{1.33}$$

It is assumed that the conditions are satisfied:

$$\forall A,\, B \in U(E) \Rightarrow A \cup B \in U(E)\,; \tag{1.34}$$

$$\forall A, B \in U\left(E\right) \Rightarrow A \setminus B \in U\left(E\right). \tag{1.35}$$

The domain of mapping *F* definition — *U*(*E*) is ring. $\varnothing \in U\,(E)$. Symmetrical difference

$$A \Delta B = \ (A \setminus B) \cup (B \setminus A) \in U(E)\,. \tag{1.36}$$

In [16] „a variation" of the set is used. A variation of the set *E* with the aid of the set *B* is called set $E' = E\Delta B$. Let further $\mu(E')$ be a measure - positive, additive, uniform function. The three (*E*, *U*(*E*), μ) is the measurable space.
The function of set *F* (*E'*) is continuous, if for $\forall \varepsilon > 0\ \exists \delta(\varepsilon, E') > 0$, such, that the condition:

$$|\, \mu(E'\Delta B) - \mu\left(E'\right) | < \delta(\varepsilon, E') \Rightarrow |\, F(E'\Delta B) - F\left(E'\right) | < \varepsilon \tag{1.37}$$

is satisfied.

Derivative *F*(*E'*) with respect to the measure μ(*E*) is determined. Let the sequence of sets $\{B_n\}$, $n \in \overline{1,\infty}$ and set *B* have such property, that for $\forall \varepsilon > 0\ \exists N(\varepsilon, \{B_n\})$, for which following condition is satisfied:

$$n > N\left(e, \left\{B_n\right\}\right) \Rightarrow |\, \mu\left(B_n\right) - \mu\left(B\right) | < \varepsilon \,. \tag{1.38}$$

Then *B* is a limit of sequence $\{B_n\}$ on the measure μ:

$$B\,|_{\mu} = \lim_{n \to \infty} B_n \quad . \tag{1.39}$$

Derivative *F*(*E'*) with respect to the measure μ is determined by the following formula:

$$\left.\frac{dF\left(E'\right)}{d\mu\left(E'\right)}\right|_{\left\{B_n\right\}} := \lim_{B_n \to B} \frac{F\left(E'\Delta B_n\right) - F\left(E'\right)}{\mu\left(E'\Delta B_n\right) - \mu\left(E'\right)} \quad . \tag{1.40}$$

If set *E'* change with a change in a certain parameter *t*, which can be interpreted as a time $t \in [t_1, t_2]$ and for $\forall t\ F' \in U(E)$, then the derivative $\frac{dF}{d\mu}$, that corresponding „to moment" *t* is determined by the relationship:

$$\left.\frac{dF}{d\mu}\right|_{t} := \lim_{B_n \to F'_t} \frac{F\left(E'_t \Delta B_n\right) - F\left(E'_t\right)}{\mu\left(E'_t \Delta B_n\right) - \mu\left(E'_t\right)} \quad . \tag{1.41}$$

1.4.4. Dynamics of sets and moments problem

The approach to the description of the dynamics of a change in the sets, given in the mentioned work [16], does not exhaust all possibilities. There are approaches based on other determinations „of distance" between the sets (depending on the nature of the studied sets): D -statistic, Manhattan distance, the distance of Minkowski, etc). It is seemed that the more detailed description

It seats that the more detailed description of the set alteration in the course of time) can be built on the basis of the approach consonant to the Hamburger moments problem (or its modifications) [4].

Let each state $\sigma_i \in S_a|\sigma_0$ can be parameterized and characterized by the real parameter $r_i \geq 0$ (for example, by some resources), and $\pi(S_a|\sigma_0)$ is the distribution of preferences on $S_a|\sigma_0$. Let's compose the sequence of sums (supposing at first that N = const):

$$l_k = \sum_{i=1}^{N} r_i^k \pi(r_i) \geq 0 \quad . \tag{1.42}$$

Hamburger proved theorem for the case of real variable $r \in (-\infty, +\infty)$. Necessary and sufficient condition such that there exist the non-decreasing function $\mu(r)$, having the finite number of points of increase and such, that

$$\int_{-\infty}^{+\infty} r^k d\mu(r) = l_k \; ; \quad \left(k \in \overline{0,\infty}\right) \quad , \tag{1.43}$$

appears the positivity of sequence $\{l_k\}$.

Positivity means that all quadratic forms for all m and any collections of real values $(x_1, x_2, \ldots, x_m)$ are non-negatively determined:

$$\sum_{i=0}^{m} \sum_{j=0}^{m} l_{i+j} \; x_i \; x_j \geq 0 \quad . \tag{1.44}$$

In our case the role of measure $\mu(r)$ plays the function of preference $\pi(r_i)$. Task simplifies by the fact that distribution $\pi(r_i)$ or, in the general case, $\pi(S_a|\sigma_0)$ is considered the given one, and the number of alternatives is finite (or countable). In any event in practice it is possible to limit only to the finite number of moments l_k; $k_{\max} = K$. Moments l_k $k \in \overline{0,K}$ form $(K+1)$ dimensional vector: $\left(l_0, l_1, l_2 \ldots, l_K\right) = \vec{l}_K \left(S_a \,|\, \sigma_0\right)$ of real positive numbers, which represents set $S_a|\sigma_0$ with the assigned on it distribution of preferences.

Now a change in the set $S_a|\sigma_0$ (with constant number of alternatives N) can be characterized as a change in the vector $\vec{l}_k$, in particular, to determine the derivative $\left.\frac{dS_a \,|\, \sigma_0}{dt}\right|_{N=\text{const}}$ as $\left.\frac{d\vec{l}_K}{dt}\right|_{N=\text{const}}$:

$$\left.\frac{dS_a \,|\, \sigma_0}{dt}\right|_{N=\text{const}} = \left.\frac{d\vec{l}_K}{dt}\right|_{N=\text{const}} \quad . \tag{1.45}$$

Let's name $\vec{l}_k$ *the k- image* of set $S_a|\sigma_0$ with the preferences. Each component of the vector $\vec{l}_k$ it is the function N of arguments r_i ($i \in 1, N$). If we introduce now the vector space L with the basis $\left(\vec{e}_0, \vec{e}_1, \ldots, \vec{e}_k\right)$, then

$$\vec{l}_K = \sum_{k=0}^{K} \vec{e}_k \; l_k . \tag{1.46}$$

Derivative

$$\frac{\partial \vec{l}_K}{\partial r_i} = \sum_{k=0}^{K} \vec{e}_k \frac{\partial l_k}{\partial r_i} = \sum_{k=0}^{K} \vec{e}_k \left(k\pi\left(r_0 \,|\, \sigma_0\right) + r_i \frac{\partial \pi\left(r_i \,|\, \sigma_0\right)}{\partial r_i} \right) r_i^{k-1} \quad . \tag{1.47}$$

If $r_0 = \sigma_0$, then

$$\frac{\partial \vec{l}_K}{\partial \sigma_0} = \sum_{k=0}^{K} r_i^k \frac{\partial \pi(r_i \mid \sigma_0)}{\partial \sigma_0} \vec{e}_k \quad .$$

Further it is possible to examine the multidimensional space of resources r_i $\left(i \in \overline{1,N}\right)$ as *N-th* vector space with the basis $\left(\vec{\varepsilon}_1, \vec{\varepsilon}_2, \ldots, \vec{\varepsilon}_N\right)$, then the gradient of function l_k in the space of the resources

$$\overrightarrow{grad}_R \, l_k = \sum_{i=1}^{N} \vec{\varepsilon}_i \frac{\partial l_k}{\partial r_i} .$$

If we designate

$$\frac{d\vec{r}}{dt} = \sum_{i=1}^{N} \vec{\varepsilon}_i \frac{dr_i}{dt}. \tag{1.48}$$

and introduce the scalar product

$$\frac{dl_k}{dt} = \overrightarrow{grad}_R \, l_k \frac{d\vec{r}}{dt} = \sum_{i=1}^{n} \frac{\partial l_k}{\partial r_i} \frac{dr_i}{dt},$$

then

$$\left. \frac{d\vec{l}_k}{dt} \right|_{N=\text{const}} = \sum_{k=0}^{K} \vec{l}_k \frac{dl_k}{dt} = \sum_{k=0}^{K} \vec{l}_k \sum_{i=1}^{N} \frac{\partial l_k}{\partial r_i} \frac{dr_i}{dt}. \tag{1.49}$$

The outlined approach to the description of dynamics of the sets change has some advantages in comparison with the approach, presented in the work [16].

1.4.5. Different sets of states

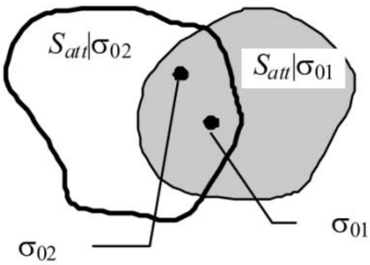


Figure 1.8

At each moment of time the system is found in one of the states, let this state is σ_0. Having the available specific resources, the subject of system can principally convert system to any state, which belongs to a certain subset $S_{att}|\sigma_0$ of attainable states.

Set $S_{att}|\sigma_0$ *includes all states* σ_i *of* „arrival“ *from the state* σ_0 *of* „departure“ *such, that the corresponding required resources for the realization of passage* $\sigma_0 \to \sigma_i$ $R^{req}(\sigma_i) \leq R^{disp}(\sigma_i)$. *Configuration of* $S_{att}|\sigma_0$ *depends on initial position* σ_0:

$$S_{att} \mid \sigma_{01}\sigma_{02} = S_{att} \mid \sigma_{01} \cap S_{att} \mid \sigma_{02} .$$

Name „boundary" state σ_{ib} such state, which corresponds to the optimal strategy of the resources use in the course of realization of the passage based on σ_0. In a general sense the corresponding variation problem relates to the class of the tasks of optimal control with the limited resources (for example [36,148]).
It is obvious that $S_{att}|\sigma_0$ is a subset of set *SA*:

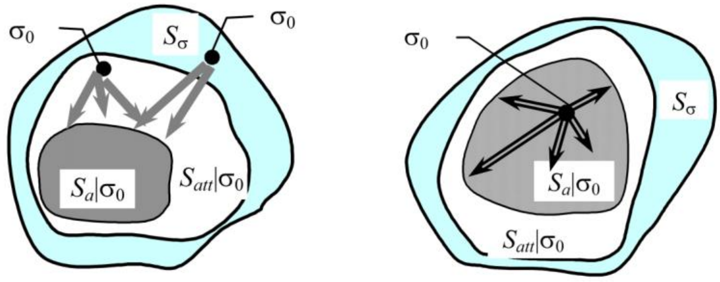
$$S_{\text{att}} \,|\, \sigma_0 \subseteq S_\sigma \,.$$

Not all states, which belong to $S_{att}|\sigma_0$, are considered by the subject as alternatives. Being located in „the position" σ_0, subject studies the part of the states of set $S_{att}|\sigma_0$ and evaluates them from the point of view of their preferability with respect to σ_0. In other words, subject introduces the relation „of the preference" ρ on the certain subset $S_a|\sigma_0 \subseteq S_{att}|\sigma_0$.Relation ρ separate from $S_{att}|\sigma_0$ subset of elements $\sigma_i \in S_a|\sigma_0$ connected with this relation. In this sense ρ is identified with the set $\rho = S_a|\sigma_0 \times S_a|\sigma_0$, where the sign „$\times$ "indicates the Cartesian product of sets. Relation ρ is a subset of the ordered pairs (σ_i, σ_j).

It is necessary to assume that all or the part of the states $\sigma_i \in S_{att}$ are distinguishible and comparable between themselves. This means that between the elements can be determined the binary relation of preference ρ. Set of states $\sigma_i \in S_{att}|\sigma_0$, between which the subject establishes the preference relation ρ: $\rangle$ (or $\underset{\sim}{\langle}$) there is set of alternatives $S_a|\sigma_0$. The states, distinguishable and comparable in ρ, that interest subject, are called alternatives.
There exist two possibilities:
1. $\sigma_0 \in S_a \,|\, \sigma_0$;
2. $\sigma_0 \overline{\in} S_a \,|\, \sigma_0$.

In the first case the subject studies set of alternatives $S_a|\sigma_0$ „from without" (Fig. 1.9, a). This means that the subject cannot remain in the initial state σ_0. This state is not one of the alternatives.

In the second case the subject „looks" on the set $S_a|\sigma_0$ „from within", and $\sigma_0 \in S_a|\sigma_0$ is one of the alternatives, i.e., subject can, in particular, select as most preferable against the background of existing alternatives the state, in which he has already been located , i.e., σ_0.



a *b*

Figure 1.9

„System" must be individualized, at least, during a certain finite time interval. In other words, it would be desirable to assume that during this interval there is a certain invariant, which characterizes system (in the sense of the theory of categories). Invariant is a function of cardinal.

If it is limited by the finite number of states either alternatives or problems, then invariant is expressed as this quantity.

We already said above that the number of alternatives in the set $S_a^{N_a} \,|\, \sigma_0$ can be changed in the course of time and, therefore, „invariant" connected with the number of alternatives is „relative". Let's name „absolute" invariant the invariant, assigned on the set $S_\sigma^{N_\sigma}$ - cardinal number („Card"):

$$I_{S_a} = \text{Card}\, H\left(S_\sigma, S_\sigma\right). \qquad (1.50)$$

Here $H(S_\sigma, S_\sigma)$ is dimorphism's set $S_\sigma \to S_\sigma$. In practice we frequently deal with respect to the systems, the number of permissible states of which can be changed in the course of time. This situation, for example, occurs when in the system any „failures" occur.
Invariant I_{S_σ} could be considered as an objective system characteristic.

„*Relative*" or subjective invariant

$$I_{S_a} = \text{Card}\, H\left(S_a \mid \sigma_0, S_a \mid \sigma_0\right) \tag{1.51}$$

is connected with the quantity of alternatives N_a, which, obviously, also changes (in the general case) in the course of time.

At first in the simplified version the systems for which N_σ = const and N_a = const, at least, during the period of time spent on the solution of the selected problem will be examined. The numbers N_σ and N_a can be changed upon transfer to the following cycle. It is possible to use other invariants, for example:

$$\text{Card}\, H\left(S_\sigma, S_\sigma\right) > \text{Card}\, H\left(S_a \mid \sigma_0, S_a \mid \sigma_0\right),$$

$$I_{S_a, R_a} = \text{Card} H\left(S_a, \mathfrak{R}_a^{req}, \mathfrak{R}_a^{\exp}\right), \tag{1.52}$$

where $H\left(S_a, \mathfrak{R}_a^{req}, \mathfrak{R}_a^{\exp}\right)$ - the set of morphisms $S_a \to \mathfrak{R}_a^{req}, \mathfrak{R}_a^{\exp}$, and $\mathfrak{R}^{req}$, $\mathfrak{R}^{\exp}$ are sets of resources.

Using the concepts introduced earlier it is possible to propose the following determination of „*intellectual catastrophe*".

Let S_σ is set of the states of the system (it can be finite, countable, continuous); $S_a|\sigma_0$ is set of alternatives, generated by subject. Cardinal numbers for S_σ and $S_a|\sigma_o$ are correlated as follows

$$\text{Card}\, H\left(S_\sigma, S_\sigma\right) > \text{Card}\, H\left(S_a \mid \sigma_0, S_a \mid \sigma_0\right) \quad ,$$

following possibilities can occur:

1. $S_{att}|\sigma_0 \subseteq S_\sigma$ and $S_a|\sigma_0 \cap S_{att}|\sigma_0 = S_a$.

Subject desires some permissible and accessible (Fig. 1.10, *a)*.

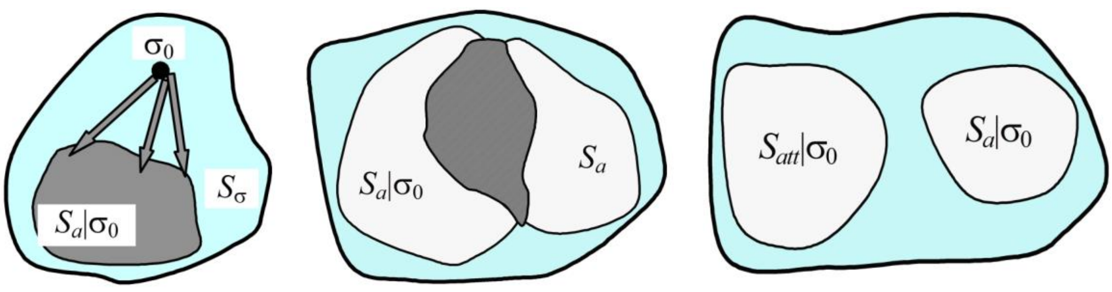
2. $S_{att}|\sigma_0 \subseteq S_\sigma$ and $S_a|\sigma_0 \setminus S_{att}|\sigma_0 \neq \varnothing$, $S_{att}|\sigma_0 \subseteq S_\sigma$.

Subject desires some permissible, but not all is accessible (Fig. 1.10, *b)*.

3. $S_{atta}| \subseteq S_\sigma$ and $S_a|\sigma_0 \cap S_{att}|\sigma_0 = \varnothing$.

Subject desires some permissible $S_a|\sigma_0 \subseteq S_\sigma$, but not accessible from σ_0 (Fig. 1.10, *c)*.

4. $S_a|\sigma_0 \setminus S_\sigma \neq \varnothing$.



a *b* *c*

Figure 1.10

Subject desires some inadmissible (not all is possible).

If $S_a|\sigma_0 \cap S_\sigma = \varnothing$, subject desires are impossible.

Finally, set of alternatives can be empty $S_a|\sigma_0 = \varnothing$. In the latter case, and also in the case 4 it is possible conditionally to speak about „*intellectual catastrophe*".

Subject does not examine any alternatives or he examines the ephemeral alternatives, which foresee reaching states not compatible with existence of this system.

If only alternative is examined, then the entropy, which corresponds to relative invariant is equal to zero (singular case), more precisely: $H_\pi = 0$, if $S_a|\sigma_0 = \sigma_1$.

„*Resource catastrophe*" appears, if for all $\sigma_i \in S_a|\sigma_0$ the following condition is satisfied

$$R^{\mathrm{req}}(\sigma_i) \geq R^{\mathrm{disp}} \quad ,$$

i.e., problems *are insolvable*.

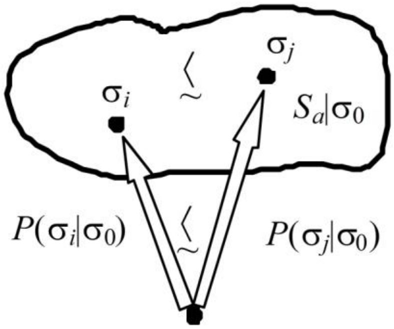


Figure 1.11

What is difference between the concepts „*problem*" and „*alternative*"? Set of alternatives $S_a|\sigma_0$are a set of the states, considered by the subject as permissible, accessible and preferable with respect to the initial state σ_0 (that occurs at the given moment). States accessible but less preferable than σ_0: $\sigma_i \precsim \sigma_0$ either are not included in $S_a|\sigma_0$ or can be included with zero utility by the subject as permissible. On the formed set $S_a|\sigma_0$ the binary relations of preference between all states are established. In this case there is a homomorphism

$$\sigma_i \precsim \sigma_j \Leftarrow \; P\left(\sigma_i \mid \sigma_0\right) \lesssim P\left(\sigma_j \mid \sigma_0\right) \quad . \tag{1.53}$$

The right side of this relationship means that the solution of problem $P(\sigma_i|\sigma_0)$ is more preferable than the solution of problem $P(\sigma_i|\sigma_0)$. Correspondence is homomorphism since the relation $\sigma_i \lesssim \sigma_j$ can occur also for other problems and another initial state σ_0.

Correspondence (1.53) shows the difference between the concepts „problem" and „alternative", and also the preference relation ρ on $S_a|\sigma_0$.

Continuing to discuss the concept of an „*active system*" we now can indicate that this is the system, which possesses the enumerated distinctive properties. In contrast to the active system, passive system, having a set of possible states S_σ, does not form set of alternatives, $S_a|\sigma_0$, it does not establish in this set system of preferences, and it does not generate „its problems".

Examples of the active systems: nuclear plant + personnel, automobile + driver, flight vehicle + crew, research laboratory + scientific leader; instructor + student + real -technical resources of instruction, computer + researcher...

Examples of the passive systems: atomic nucleus, computer, automobile, the solar system, the atmosphere, ocean,...

By analogy with „thermodynamic death" let's name „*entropy death*" of active system the situation, when: (a) resource (including - information) exchange with „the environment" is absent and for $\forall\sigma_i \in S_\sigma$ the condition $R^{\text{req}}(\sigma_i) = idem(i)$ is satisfied, (*b)* entropy $H_\pi = H_{\pi\max}$ and this situation is steady (definition of subjective entropy see in chapter 3 of this works).

A change of system state occurs as a result

- of the goal-directed activity, connected with the expense of the available resources;
- of the changes, proceeding in „the environment", not depending on the system, but changing its state;
- „spontaneous" change in the system as a result „internal" processes, proceeding in the system.

It would be tempting to interpret all these changes as changes *in the available resources*. This method of formalization makes the theory more laconic and makes it possible to significantly unify mathematical description. This however does not always correspond to the essence of the matter, especially, if we consider influence on the distribution of the preferences of ethical factors.

1.4.6. Properties of simple alternatives

Let's return to the properties of alternatives. Alternatives σ_i and σ_j are incompatible, if they cannot be realized simultaneously. For example, subject cannot simultaneously be located in two different places; it is not possible on the competitions to occupy simultaneously the first and second places for one athlete. Let's designate this circumstance by the symbol

$$\sigma_i \wedge \sigma_j = \varnothing ,$$

where $\wedge$ is the composition of alternatives. Such alternatives can be achieved consecutively; in particular they can be trajectory elements (track):

$$\ldots \to \sigma_i \to s_j \to \ldots$$

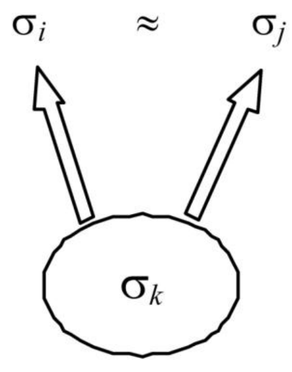


Figure 1.12

Together with „the composition" $\sigma_i \wedge \sigma_j$ (binary composition) the binary „disposition" $\sigma_i \vee \sigma_j = \sigma_s$ can be used alternative consisting in the fact that one of three possibilities: σ_i either σ_j or simultaneously and σ_i and σ_j (i.e., composition) will be realized.

Two alternatives are independent, if two ways Tr_{ij}: $\sigma_i \to \sigma_j$ and Tr_{ji}: $\sigma_j \to \sigma_i$ are equivalent. This means that the realization of each of the alternatives does not assume the obligatory realization of another: $Tr_{ij} \sim Tr_{ji}$. Two alternatives σ_i and σ_j are equivalent conditionally relative to the third alternative σ_k, if the way Tr_{kj} and Tr_{ki} are equivalent.

The classes (subsets) of equivalence have already been discussed above. Each element $\sigma_i \in S_a$ has its class of equivalence, which contains as minimum one element σ_i. Designate through $S_{ai} \subseteq S_a$ the class of equivalence of element σ_i .. Set S_a thus is separated on the nonintersecting classes of equivalence. If two elements σ_i and σ_j are equivalent: $\sigma_i \sim \sigma_j$, then their classes of equivalence coincide $S_{ai} = S_{aj}$. The function of preferences is constant on any class of equivalence (this circumstance has already been used above):

$$S_{ai}\ \pi(\sigma_i) \rightarrow idem(i); \quad \forall\sigma_i \in S_{ai}$$

If on S_a preference relation ρ: 〉 is assigned, and S_{ai} is a class of the equivalence of element σ_i (as this element any element $\sigma_i \in S_{ai}$ can be selected), then the remaining part of the complete set S_a: $S_a \backslash S_{ai}$ is divided an two subsets S^+_{ai} and S^-_{ai} — more preferable than σ_i and less preferable elements than σ_i . If σ is the parameter with the values on real axis, then on the basis the theorem about the fact that the open set on real axis is a sum of the finite or countable number of mutually disjoined intervals (open subsets), we will consider also for *the n-dimensional* case that the classes of equivalence are the open subsets. If set S_a is finite or countable, then „boundary" between S^+_{ai} and S^-_{ai} resembles the boundary between the states, surrounded as on two sides „boundary posts" - by the alternatives, which belong to the co-sets of equivalence.

Assume that the following binary relations $\sigma_i \rangle \sigma_k$; $\sigma_j \rangle \sigma_k$, are now determined, moreover it is previously known that from the realization of problem P: $(\sigma_k \rightarrow \sigma_i)$ follows the realization of problem P: $(\sigma_k \rightarrow \sigma_j)$. In this case, we will indicate that the alternative σ_i " *absorbs*" alternative σ_j: $\sigma_i \rangle\rangle \sigma_j|\sigma_k$. This means that „the alternative" σ_i absorbs alternative σ_j with the condition that, the initial and directly previous state is state σ_k. The property of absorption, does not coincide with the properties of inclusion or belonging ($\subset$, $\in$). Each element σ_i has its absorbed set, which in particular case contains only one element σ_i. In the case of absorbing the alternatives we will say that problem P: $(\sigma_k \rightarrow \sigma_i)$ *absorbs* problem P: $(\sigma_k \rightarrow \sigma_j)$. This does not mean that the alternative σ_j can be excluded from the examination.

For example, if available resources are insufficient for the solution of the first problem, then it is necessary to solve second problem. The property of absorption is transitive, reflexive and asymmetric.

States σ_i and σ_j are incompatible, if there does not exist such a state $\sigma_k \in S_a$, which would absorb simultaneously σ_i and σ_j with the initial state σ_k and they are compatible, if such a state exists.

In the first case the state $\sigma_s \sim \sigma_i \wedge \sigma_j$ is a not realized state in S_a: $\sigma_s \overline{\in} S_a$, in the second case $\sigma_s \sim \sigma_i \wedge \sigma_j \in S_a$. Recalling the determination of the states composition, name problem $P_s \sim P_i \wedge P_j$ *„the product"* of problems P_i and P_j, if P_s: $(\sigma_k \rightarrow \sigma_s)$, where $\sigma_s \sim \sigma_i \wedge \sigma_j \backslash \sigma_k$. Problem P_s *„is solvable"* in S_a, if $\sigma_s \in S_a$ and *„is insoluble"* in S_a if $\sigma_s \overline{\in} S_a$.

This, the problem, which provides as a desired alternative the composition of alternatives, is the product of problems. Let's designate, through $P_s = P_i \vee P_j$ *„sum"* of problems P_i and P_j — the problem, which supposes reaching at least one of the alternative states: σ_i or σ_j. from the initial state σ_k. Problem P_s is solvable, if at least one of the problems P_i and P_j is solvable.

The set of the elements, for each of which the subset of absorption consists of this one element, name *fundamental*, on the contrary, if all states are built in a row of the mutually-absorbing states

$$\sigma_1 \langle\langle \sigma_2 \langle\langle \sigma_3 \langle\langle \ldots \langle\langle \sigma_{N-1} \langle\langle \sigma_N\ ,$$

corresponded set name universal or *universally- connected*.

Such a situation can occur, if all elements are characterized by one universal index (for example, by cost equivalent), and subject possesses the necessary quantity of units of this equivalent in order to realize alternative σ_{i+1}. In this case alternative σ_i will be realized and also all *„young"* alternatives σ_{i-1}, σ_{i-2},...

As an example can serve collection as the alternatives of points, consecutive on the track (Fig. 1.13). Reaching of A_i point includes reaching of all preceding points.

Let's give some additional definitions, which are concerned category *„problem"*:

1. „*Simplest problem*" $P^{(1)}$: $(\sigma_0 - \sigma_i)$ — the problem, whose solution is the one-act action of passage from the initial state σ_0 to the another, „simple" state σ_i.

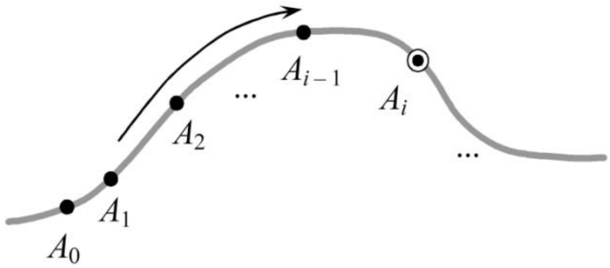


Fig.1.13

2. „*Complex problem*" $P^{(k)}$:$(P_1, P_2, ..., P_k) \sim \{(\sigma_0 \to \sigma_1), (\sigma_0 \to \sigma_2), ..., (\sigma_0 \to \sigma_k)\}$ is the collection of the simple problems, based on which in the final analysis a certain composition (or product) of order $n \le k$ is selected. Complex problem can consist not only of simple problems, but also include different compositions. Thus, if S_a is countable, then set of all complex problems have a power of continuum.
3. „*Vector problem*" $\vec{P}_s$ - corresponds to that case, when the desired state σ_i is represented in the form the collection of components and can be considered as the vector

$$\vec{\sigma}_i = \left\{\sigma_i^1, \sigma_i^2, ..., \sigma_i^s\right\}.$$

Then problem, also is represented as the vector $\vec{P}_s : \left\{P_i^1, P_i^2, ..., P_i^s\right\}$. This means that there is a certain linear transformation in the *s-dimensional* space, components of which they are variable $\sigma_i^r \left(r \in \overline{1,s}\right)$.

4. „*Hierarchical problem*" - is the problem, formed as the collection of the simple (or complex) problems, deciding in the assigned sequence. Moreover so that the solution of each subsequent problem is possible only after the solution of previous one, there is a resource conditionality of this sequence.
5. „*The subordination*" of problems - is relationship between the problems, when a certain sequence of the solution of problems is established, but there is no resource conditionality of this sequence.